CHAPTER 12

Fractions, Decimals, Ratios, and Percents

Goals

You will be able to

- relate and compare fractions, decimals, and percents
- identify, model, and apply ratios in various situations
- represent relationships using unit rates
- solve problems using a guess and test strategy

Experimenting with a globe

Water	Desert
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Getting Started

You will need

- counters
- grid paper
- pencil crayons

Filling a Pancake Order

Kurt's class is having a pancake breakfast. Each student ordered one type of pancake in advance.

$\frac{1}{2}$ of the students ordered chocolate chip, $\frac{1}{6}$ of the students ordered blueberry, $\frac{1}{4}$ of the students ordered plain, and the rest ordered strawberry.

? What fraction of the class ordered strawberry pancakes?

A. Could there be 16 students in the class? Could there be 26 students in the class? How do you know?

B. What do the fractions of students who ordered each type of pancake tell you about the class size?

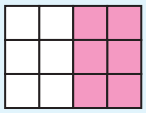
C. Determine a possible class size. Show your work.

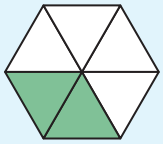
D. For the class size you chose in Part C, determine the number of each kind of pancake they need to make.

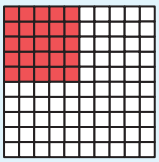
E. What fraction of the class ordered strawberry pancakes?

Do You Remember?

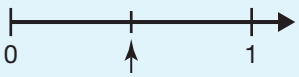
1. What fraction of each whole is coloured?
Name an equivalent fraction for each picture.

a) 

b) 

c) 

2. Match each number on the left with an equivalent representation on the right.

a)	0.4	**A**	
b)	0.2	**B**	
c)	$\frac{5}{9}$	**C**	four tenths
d)	$\frac{1}{4}$	**D**	
e)	$\frac{3}{2}$	**E**	
f)	0.5	**F**	
g)	$1\frac{1}{4}$	**G**	1.5

3. Compare. Write $>$, $<$, or $=$. Explain your choice.

a) $\frac{7}{4} \blacksquare \frac{3}{4}$ **c)** $\frac{2}{5} \blacksquare \frac{4}{10}$ **e)** $\frac{11}{5} \blacksquare \frac{2}{5}$

b) $\frac{4}{10} \blacksquare \frac{7}{10}$ **d)** $\frac{9}{10} \blacksquare 1\frac{1}{10}$ **f)** $\frac{6}{12} \blacksquare \frac{1}{4}$

4. Write a decimal equivalent for each. Show your work.

a) $\frac{4}{5}$ **b)** $\frac{7}{10}$ **c)** $3\frac{1}{2}$ **d)** $\frac{5}{2}$

5. Write an equivalent fraction for each.

a) 0.73 **b)** 0.25 **c)** 0.6 **d)** 1.5

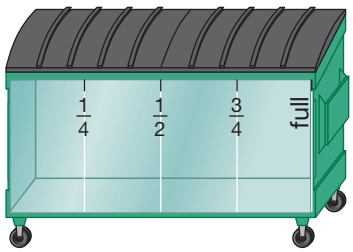
1 Comparing and Ordering Fractions

You will need
- fraction strips
- number lines

Goal **Compare and order fractions on number lines.**

All families in Meadowcreek are supplied with containers that hold four bags of garbage each. The weekly limit for free pick-up is one container. Every bag over the limit costs $2.00.

Jorge is in charge of the family garbage.

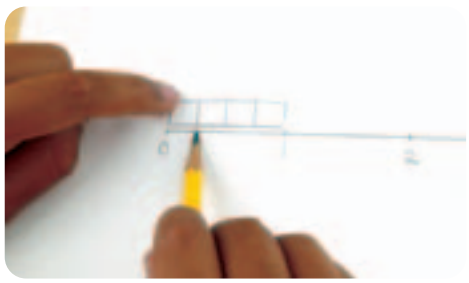


? How much will Jorge's family pay for garbage pick-up each week?

Jorge's Solution

I'll place the number of containers for each week on a number line.

Since four bags represent one full container, I need to divide the line into fourths. A fraction strip will help.



This Month's Garbage

Week	Number of filled containers
Week 1	1
Week 2	$\frac{3}{4}$
Week 3	$2\frac{1}{2}$
Week 4	$\frac{6}{4}$

A. How long should the number line be to show the number of containers for each of the four weeks?

B. Draw Jorge's number line. Label the number of containers for each week.

C. List the fractions in order from least to greatest.

D. In which weeks did Jorge's family fill more containers than in Week 1? Compare using an **inequality sign**.

E. How many bags over the limit did Jorge's family put out each week?

F. How much will Jorge's family pay for garbage pickup each week?

Reflecting

1. How did you decide where to put the **improper fraction** $\frac{6}{4}$?
2. Explain your strategy for ordering the numbers.

Checking

3. Jorge's family makes liquid fertilizer from their fruit and vegetable waste.
   a) Which fraction strip will help you draw a number line with a place for each number?
   b) Use an inequality sign to compare the amounts made in Week 1 and Week 2.
   c) List the amounts of fertilizer from least to greatest.

Week	Amount of fertilizer (buckets)
Week 1	$1\frac{1}{5}$
Week 2	$\frac{7}{5}$
Week 3	$\frac{3}{5}$
Week 4	$2\frac{2}{5}$

Practising

4. Compare. Write $>$, $<$, or $=$. Explain your strategy.
   a) $\frac{5}{4} \blacksquare \frac{6}{4}$ b) $\frac{3}{2} \blacksquare 3\frac{1}{2}$ c) $1\frac{1}{2} \blacksquare 1\frac{3}{4}$ d) $\frac{2}{5} \blacksquare \frac{3}{10}$
5. Order each set of numbers from least to greatest.
   a) $\frac{3}{4}, \frac{4}{2}, 2\frac{1}{2}, \frac{7}{4}$ b) $2\frac{8}{10}, \frac{6}{5}, \frac{9}{5}, \frac{9}{10}, 1\frac{1}{10}$
6. The chart shows hours spent doing weekend chores.
   a) Which fraction strip will help you draw a number line with a place for each number in the chart? Explain your thinking.
   b) Use an inequality symbol to compare the number of hours Chris and Gabriel spent on chores.
   c) Order the fractions in the chart from least to greatest.

Name	Time to complete chores (in hours)
Mia	$\frac{5}{4}$
Chris	$\frac{9}{4}$
Katherine	$1\frac{1}{2}$
Andrea	$\frac{3}{4}$
Gabriel	$1\frac{3}{4}$

2 Comparing Fractions with Unlike Denominators

You will need
- grid paper
- fraction strips
- a calculator

Goal **Compare fractions when the denominators are different.**

The 15 km Charity Walk-a-thon has
- a trail mix station every two thirds of a kilometre;
- a water station every three fourths of a kilometre; and
- a cooling station every three halves of a kilometre.

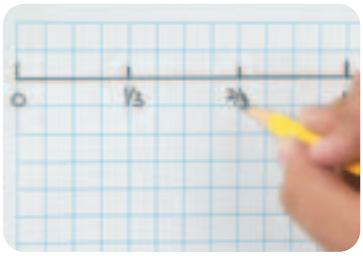
Ayan has reached the first water station, Mark is at the first cooling station, and Angele is at the second trail mix station.

? Who has walked the farthest?

Raven's Solution

I'll make a sketch of the first 2 km of the race on grid paper and mark the stations. I need to show halves, thirds, and fourths, so I want a whole that I can easily divide by 2, 3, or 4. I think a whole with 12 sections will work. 1 km will be represented by 12 squares.

A third of 12 squares is 4 squares, so $\frac{1}{3}$ is 4 squares past 0 and $\frac{2}{3}$ is another 4 squares past $\frac{1}{3}$.



A. Draw a number line for 0 km to 2 km. Mark all of the thirds. Label the trail mix stations.

B. Mark the fractions for the cooling stations.

C. Mark the fractions for the water stations.

D. Mark the locations of the three students.

E. Who has walked the farthest?

Reflecting

1. How did you decide where to put the water and cooling stations?
2. Look at the positions of these fractions on the number line: $\frac{2}{2}$, $\frac{2}{3}$, and $\frac{2}{4}$. Which fraction is greatest? Why does that make sense?
3. How can you compare fractions when both the numerators and denominators are different? Use $\frac{3}{4}$ and $\frac{2}{3}$ as an example.

Checking

4. a) Write fractions for the locations of these stations:
   b) Order the fractions in Part a) from greatest to least.
   c) Explain the strategies you used to order the fractions.

- T, the fourth trail mix station
- W, the third water station
- C, the third cooling station

Practising

5. Compare. Write $>$, $<$, or $=$. Explain your strategy.
   a) $\frac{5}{6} \blacksquare \frac{1}{6}$ b) $\frac{2}{4} \blacksquare \frac{2}{5}$ c) $1\frac{1}{2} \blacksquare \frac{3}{4}$ d) $\frac{5}{2} \blacksquare 3\frac{1}{2}$
6. For each pair of stations in the Charity Walk-a-thon, which is farther from the start of the race? How do you know?
   a) The second water station or the third trail mix station
   b) The third cooling station or the fourth water station
   c) The sixth trail mix station or the fourth cooling station
7. For each pair of chores, which one took longer to complete? Tell how you know.
   a) $\frac{4}{5}$ h doing laundry or $\frac{2}{5}$ h vacuuming
   b) $\frac{1}{3}$ h washing dishes or $\frac{1}{5}$ h drying dishes
   c) $\frac{1}{2}$ h collecting garbage or $\frac{3}{5}$ h cleaning the bathroom
8. Count by fourths from 0 to 4.
9. What is the greatest value you can use to make each true?
   a) $\frac{\blacksquare}{5} < \frac{3}{4}$ b) $3\frac{2}{3} < 3\frac{4}{\blacksquare}$ c) $4\frac{3}{8} > \blacksquare\frac{2}{3}$

3 Fraction and Decimal Equivalents

You will need

- fraction strips
- number lines
- play money
- a calculator

Goal **Relate fractions to decimals and determine equivalents.**

Tara wrote a story and removed the numbers to create a puzzle.

> Maggie needs $■ for a one-way bus fare to visit her Grandma. In her piggy bank she only has $■, which is ■ of a dollar. She asks her Mom for $■ to make up the difference. Another name for that amount is ■ of a dollar. If Maggie were to return home on the bus as well, she would need a total of ■ of a dollar.

These are the numbers from Tara's story:

$\frac{2}{5}$, 0.90, 0.50, $\frac{18}{10}$, 0.40, $\frac{1}{2}$

? How can you make a story puzzle like Tara's?

A. Copy the story and replace each ■ with one of the numbers listed so that the story makes sense.

B. How did you decide which number to use in each spot?

C. Write your own story that uses about five fractions and decimals. Make sure you use both fractions and decimals.

D. Create a puzzle for a classmate to solve by rewriting your story with the numbers taken out and listing them in a different order.

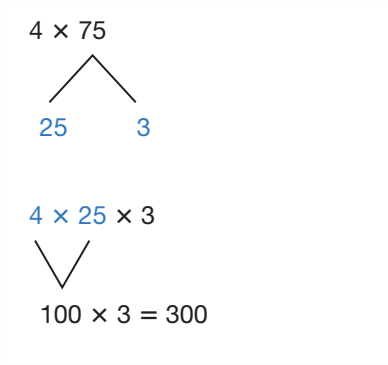
Reflecting

1. Explain how to write a fraction like $\frac{2}{5}$ as a decimal equivalent without using a calculator.
2. How do you write the decimal 0.25 as an equivalent fraction?

Using Factors to Multiply

Sometimes, you can use the factors of a number to multiply with mental math.

To calculate 4×75, list factors of 75.



A. What is another way you can use mental math to multiply 75 by 4?

Try These

1. Calculate.

a) 4×15 **c)** 12×15 **e)** 4×75

b) 8×25 **d)** 12×25 **f)** 12×75

2. How can your answer to Question 1e) be used to determine 4×7.5?

3. Explain how knowing $4 \times 25 = 100$ can help you calculate each product.

a) 16×25 **b)** 4×250 **c)** 5×25

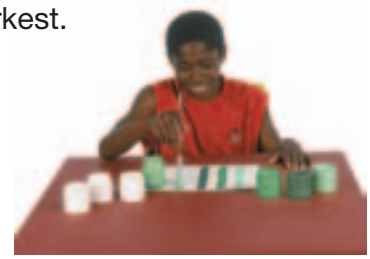
4 Ratios

You will need
- counters

Goal **Identify and model ratios to describe situations.**

Rodrigo is trying a painting technique he read about in an art book. He mixes different **ratios** of green and white paint to make four distinct tints.

He wants to order the shades of paint from lightest to darkest.



Tint	Number of containers of green paint	Number of containers of white paint
A	3	0
B	3	1
C	3	2
D	3	3

ratio
A comparison of two numbers or quantities measured in the same units.

If you mix juice using 1 can of concentrate and 3 cans of water, the ratio of concentrate to water is 1 : 3, or 1 to 3.

? What order should Rodrigo use for the tints?

Rodrigo's Solution

I will use counters to model the mixtures. I decided to write the ratios as green : white instead of white : green.

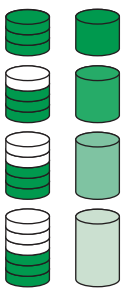
When I start, the ratio of green paint to white paint is 3 : 0.

A green : white = 3 : 0 ● ● ●

B green : white = 3 : 1 ● ● ● ○

C green : white = 3 : 2 ● ● ● ○ ○

D green : white = 3 : 3 ● ● ● ○ ○ ○



The ratio with the most white paint compared to green represents the lightest green.

From lightest to darkest I should list D, C, B, A.

Communication Tip
For ratios, read the symbol : as "to."

Ratios can be written as **part** to **part** or **part** to **whole**.

In the juice example, concentrate to water, 1 : 3, is a part to part ratio

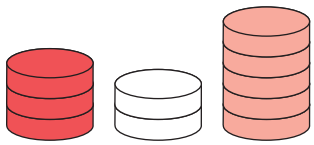
and concentrate to juice, 1 : 4, is a part to whole ratio.

Reflecting

1. Rodrigo wrote the green : white ratio for each mixture. What other part: part ratios might he have chosen to write? What part to whole ratios?
2. When you are working with a part to part ratio, how do you determine what the whole is?
3. What happened to the total amount of paint when Rodrigo added more and more white parts?

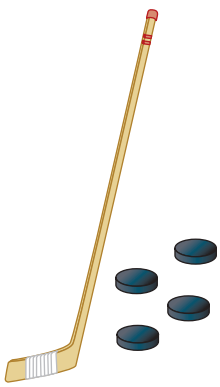
Checking

4. Rodrigo decided to make different shades of red. His first mixture used 3 cups of red paint to 2 cups of white paint.
   a) Write the ratio of the number of cups of red paint to the number of cups of white paint.
   b) Write the ratio of the number of cups of red paint to the total number of cups of paint.
   c) Write the ratio of the number of cups of white paint to the total number of cups of paint.

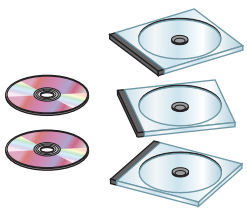


Practising

5. a) Write the ratio of the number of pucks to the number of sticks.
   b) Write the ratio of the number of sticks to the number of pucks.



6. a) Write the ratio of the number of DVDs to the number of cases.
   b) Write the ratio of the number of cases to the number of DVDs.



7. Jason is mixing lemonade from concentrate. The recipe reads, “Mix the concentrate and water in the ratio 1 to 4.”
   a) Model the ratio using counters. Sketch your model.
   b) If Jason uses 1 cup of concentrate, how much lemonade will he make?
   c) How else can you describe this mixture using part to part or part to whole ratios? Include words and numbers for each ratio.

5 Equivalent Ratios

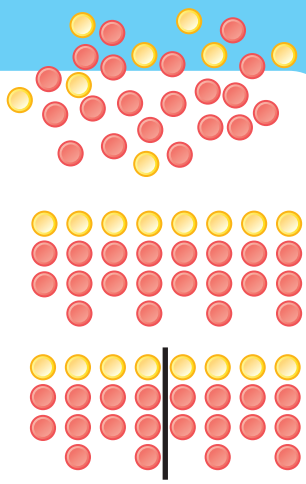
You will need
- counters
- base ten blocks

Goal **Determine equivalent ratios and use them to solve problems.**

Akeem found a collection of 28 baseball cards in the attic. eight of the cards are valuable. He decides to share the collection fairly between his four younger cousins.

? How should he divide the collection fairly?

Akeem's Solution



I will represent the eight valuable cards with yellow counters and other cards with red counters.

To be fair, I need to make smaller sets with yellow to red counters in the same ratio as in the original set. I need **equivalent ratios** for 8 : 20.

First, I will divide the cards into two sets.

To keep the comparison the same, I will split the yellow counters in half and the red counters in half.

A. Write the ratio of the number of valuable cards to the number of regular cards in each of Akeem's two new sets.

B. Why is it reasonable to say that this new ratio is equivalent to 8 : 20?

C. To make four sets, divide each of Akeem's new sets into two more sets that each have a ratio equivalent to 8 : 20. Record this ratio in a ratio table.

Number of valuable cards	8	
Number of regular cards	20	

D. How many valuable cards does each cousin get? How many regular cards does each cousin get? How many cards does each cousin get altogether?

equivalent ratios
Two or more ratios that represent the same comparison

1 : 3 2 : 6 3 : 9

Reflecting

1. How are the numbers in the ratio table related to each other?
2. Li Ming says she could have made the four equal sets by dividing each number in the original ratio by four. Do you agree or disagree? Explain.

Checking

3. Ellen has a set of stamps. The ratio of valuable stamps to regular stamps is 5 to 12.
   a) After one year her set is twice as big, but it still has the same ratio of valuable stamps to regular stamps. Compare the new number of valuable stamps to regular stamps with a ratio.
   b) Is the new ratio equivalent to the old ratio? Explain.
   c) Two years later, her stamp collection is three times as big as the original set. It still has the same ratio of valuable stamps to regular stamps. How many valuable stamps are in this set?

Practising

4. To make juice, you mix cans of concentrate with cans of water in the ratio of 2 to 6.
   a) Write an equivalent ratio of concentrate to water.
   b) Make a ratio table to represent the juice mixture.
   c) How many cans of water would you need to mix with five cans of concentrate? Explain your thinking.
5. Copy each equation. Replace the ■ with a number to make an equivalent ratio.
   a) 20 to 6 = 10 to ■
   b) 3 to 8 = 21 to ■
   c) 3 to 5 = 15 to ■
   d) 1 : 1 = ■ : 7
6. Lucia makes fruit leather to sell at a farmer's market. She uses a ratio of 6 bananas to 4 peaches.
   a) Write ratios equivalent to 6 : 4.
   b) If Lucia has 16 peaches, how many bananas does she need?
   c) If Lucia has 48 bananas, how many peaches does she need?

Bananas	3	6	9	12		
Peaches		4			10	12

Frequently Asked Questions

Q. How do you compare and order fractions?

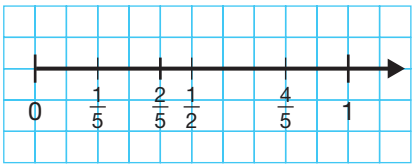
A. When the denominators are the same, a fraction with a greater numerator is greater because there are more identical parts of the same whole.

$$\frac{4}{5} > \frac{2}{5}$$

When the numerators are the same, a fraction with a greater denominator is less. It has the same number of parts, but the whole for the fraction with the greater denominator is broken up into more parts. Each part is smaller.

$$\frac{1}{5} < \frac{1}{2}$$

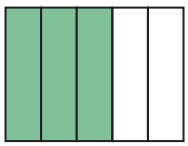
When neither the denominators nor the numerators of the fractions are the same, you might use benchmark fractions like $\frac{1}{2}$, $\frac{1}{4}$, 0 and 1 to order.



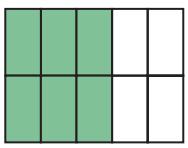
Or you might place the fractions on a number line, representing a whole by a convenient number of sections. The fraction farthest to the right is greatest.

Q. How do you write fraction and decimal equivalents?

A. Look for an equivalent fraction with a denominator of 10, 100, or 1000. Then use place value to write the decimal equivalent.



For example, $\frac{3}{5} = \frac{6}{10}$, which is read as six tenths and can be written as the decimal 0.6.

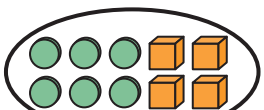


Follow place value rules to write a decimal as a fraction. Then write equivalent fractions if you want a different denominator.

For example, $0.05 = \frac{5}{100} = \frac{1}{20}$.

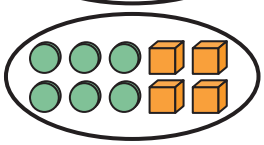
Q. How do you determine equivalent ratios?

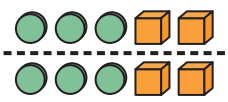
A. Equivalent ratios represent the same comparison. You can model a ratio, then make copies of the model or break the model into equal parts.



For example, consider the ratio 6 : 4.

Model the ratio and make copies of it to get
6 : 4 = 12 : 8,
or model the ratio and divide it into equal parts to get
6 : 4 = 3 : 2





LESSON

Mid-Chapter Review

1

1. Order each set of numbers from least to greatest. Show your work on a number line.

a) $2\frac{1}{4}, \frac{5}{4}, \frac{3}{4}$ **b)** $\frac{2}{5}, \frac{8}{5}, 1\frac{4}{5}$

2

2. Compare. Write $>$, $<$, or $=$. Show your work.

a) $\frac{5}{8} \blacksquare \frac{5}{12}$ **b)** $\frac{11}{4} \blacksquare \frac{5}{3}$ **c)** $1\frac{1}{4} \blacksquare \frac{8}{5}$

3

3. Write each fraction as a decimal equivalent.

a) $\frac{1}{5}$ **b)** $2\frac{1}{2}$ **c)** $\frac{5}{4}$

4. Write each decimal as an equivalent fraction.

a) 0.2 **b)** 0.06 **c)** 0.55

4

5. In a recipe for trail mix, the ratio of nuts to dried fruits is 6 : 5.

a) If Kylie uses 6 cups of nuts in the recipe, how much trail mix will she have altogether?

b) Explain what ratio 5 : 11 might represent about the trail mix.

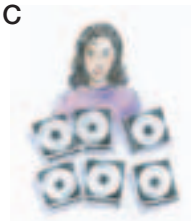
c) Write four other ratios describing the mix of nuts and dried fruit.

5

6. Which ratios of girls to CDs are equivalent?

A 

B 

C 

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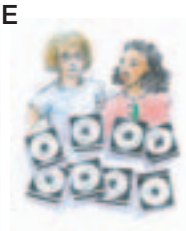
E 

F 

7. Luis made pie filling with blueberry and sugar in a 4 to 1 ratio.

a) Model the ratio using counters. Sketch your model.

b) If Luis made 5 cups of pie filling altogether, how much of each ingredient did he use?

c) Haley adds 2 extra cups of blueberries and 1 extra cup of sugar to Luis's pie filling. What can Luis do to correct the ratio of fruit to sugar?

8. Millet is a grain. You can cook millet with 3 cups of water to 1 cup of grain. How many cups of water do you need for 7 cups of grain?

6 Percents as Special Ratios

You will need
- a 10-by-10 grid
- a calculator

Goal **Understand the meaning of percent.**

James's class did a probability experiment. They threw a beach ball globe from person to person and recorded the location of the catcher's left pointer finger. This is what they recorded:

- The ratio of times the location was in the Atlantic Ocean to the total number of tosses was 4 to 25.
- The fraction of times the location was in the Pacific Ocean was $\frac{7}{20}$.

? According to the experiment, which ocean covers more of the Earth's surface?

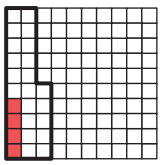
percent
A part-to-whole ratio that compares a number or an amount to 100

$25\% = 25 : 100 = \frac{25}{100}$

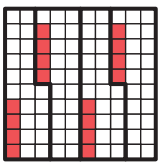
James's Solution

I need to compare the ratio 4 to 25 and the fraction $\frac{7}{20}$.
If I write them both as **percents** I will be comparing to the same whole, 100.

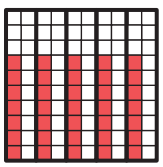
To write the ratio 4 : 25 as a percent, I need to determine an equivalent ratio with 100 as the second number.
I can represent 4 : 25 on a hundredths grid. I'll outline 25 squares and colour 4 of them.



I can keep outlining 25 squares and colouring four until the whole grid is outlined.
16 squares are coloured. 4 : 25 is equivalent to 16 : 100.
The finger was on the Atlantic Ocean 16% of the time.



I can write 7 out of 20 as either $\frac{7}{20}$ or the ratio 7 : 20.
I can write an equivalent percent using the hundredths grid.
7 : 20 is equivalent to 35 : 100.
The finger was on the Pacific Ocean 35% of the time.



According to the experiment, the Pacific Ocean covers more of the Earth's surface than the Atlantic Ocean.

Reflecting

1. How was the grid useful in determining each percent?
2. How did writing the ratio 4 to 25 and the fraction $\frac{7}{20}$ as percents allow James to compare the results?
3. Explain how to write a ratio or fraction as a percent. Use examples.

Communication Tip
Percents are written with a percent sign (%).

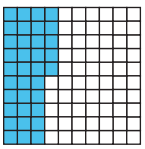
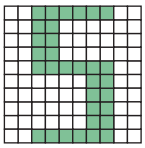
The percent sign is like writing "of each 100." 25% is read "25 percent" and means "25 of each 100"

100% means the whole.

Checking

4. James researched these numbers about drylands. Canada has 3 km^2 of drylands for each 20 km^2 of total land area.
   a) Write an equivalent ratio with 100 as the whole.
   b) Write the percent of dryland in Canada.

Practising

5. Write each as a ratio, a fraction, and a percent.
   a) 12 to 100 b) $\frac{91}{100}$ c) 0.01 d) 50 out of 100
6. For each part to whole ratio, write an equivalent ratio using 100 as the whole. Use a hundredths grid. Write each ratio as a percent.
   a) 1 to 2 b) 1 : 4 c) 6 to 25 d) 18 to 20
7. What percent of each whole is coloured?
   a) 
   b) 
   c) 
8. The population of teachers and students in a school is 4% teachers.
   a) Write a ratio for the number of teachers to the population.
   b) Write a ratio for the number of students to the population.
   c) Write a ratio for students to teachers.
9. Statistics Canada found that 22% of students chose baseball as their favourite sport. Faith surveyed the 25 students in her class. 12 chose baseball. Is baseball more or less popular in Faith's class than in Canada? Show your work.
10. Create and solve your own question that involves percents.

7 Relating Percents to Decimals and Fractions

You will need
- a calculator
- a 10-by-10 grid

Goal **Compare and order percents, fractions, and decimals.**

Rodrigo gathered newspaper clippings about the materials in a local landfill site. One article was damaged so he couldn't read part of it.

? What material takes up the most space at the landfill site?

Rodrigo's Comparison

Writing everything as a percent will make it easier to compare the numbers. I'll make a table and include a column for decimals to help me go from fractions to percents.

In the first row, 1 out of 4 is equivalent to 25 out of 100, which is 0.25 or 25%.

Material	Percent	Fraction	Decimal
Plastic		$\frac{1}{4}$	
Food and yard waste	11%		
Miscellaneous trash		$\frac{1}{5}$	
Paper	30%		
Rubber and leather	? (2% more than metal)		
Metal			
Total			

A. If everything from the landfill site is included in the table, what should be the total for the percent column? What should be the total for the decimal column?

B. Copy the table and complete the first four rows.

C. How much of the landfill is filled with rubber, leather, and metal altogether?

D. How much of the landfill is filled with rubber and leather? How much is filled with metal? Complete the table.

E. Order the materials from least amount to greatest.

F. What material takes up the most space at the landfill site?

Reflecting

1. Explain how you knew the totals for the percent column and the decimal column.
2. a) How do you write a fraction as a percent? Use $\frac{6}{8}$ as an example.
   b) How do you write a decimal as a percent? Use 0.3 as an example.
3. Did you order the amounts using the percent, fraction, or decimal column? Explain.

Checking

4. The Scouts are collecting paper for a recycling drive.
   a) Copy and complete the table.
   b) Order the types of paper from least to greatest.

Type of Paper	Percent	Fraction	Decimal
Newspaper		$\frac{1}{2}$	
Magazines	40%		
Other			0.1
Totals			

Practising

5. Write each set of numbers as percents and order them from least to greatest.
   a) 0.4, 0.04, 0.44, 0.1
   b) $\frac{1}{2}$, $\frac{47}{100}$, $\frac{3}{10}$
   c) 0.75, $\frac{1}{4}$, $\frac{3}{5}$, 0.85

6. Hannah researched different countries to see what part of the world's rainforest each country has.
   a) Copy and complete the table.
   b) Order the locations from least amount of rainforest to most.

Location	Fraction	Decimal	Percent
Brazil	$\frac{8}{25}$		
Indonesia		0.11	
Zaire			9%
Other			
Total			

7. The Grade 5, 6, and 7 classes are putting on a talent show. Fifty percent of the participants are from Grade 6 and 0.3 of the participants are from Grade 5. What is the fraction of participants from Grade 7?

8. About 65% of last year's campers are returning this year. Which of these statements is more accurate? Explain your choice.

All but $\frac{3}{8}$ of last year's campers are returning this year.

About $\frac{1}{4}$ of last year's campers are not returning this year.

8 Estimating and Calculating Percents

You will need
- a calculator
- a 10-by-10 grid

Goal **Estimate and calculate percents.**

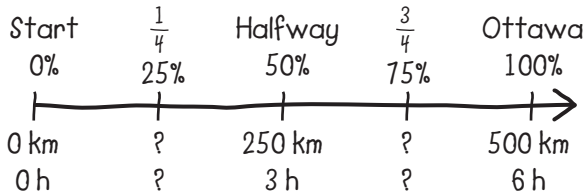
The Grade 6 class is on a 500 km bus trip to Ottawa.
It will take about 6 h.
"How much farther till we're there?" asked Isabella.
"How many more hours till we're there?" asked Khaled.

The bus driver replied, "We've gone 25% of the way."

? How many kilometres are left in the trip and how long will it take to travel that far?

Chandra's Solution

100% of the trip is 500 km. 100% of the trip will take 6 h.
I can draw a number line to help me answer the questions.



50% of the trip is the same as $\frac{1}{2}$ of the trip.
One half of 500 km is 250 km. One half of 6 h is 3 h.

A. Draw a number line like Chandra's. Complete the distances for 25% of the way and 75% of the way.

B. Complete the times for 25% of the way and 75% of the way.

C. How many kilometres do they have left to go?

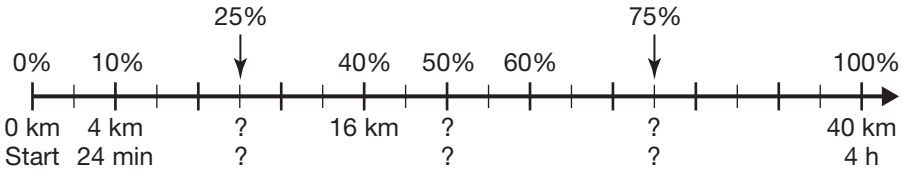
D. How many more hours will the bus trip take?

Reflecting

1. What strategy did you use to determine 25% of the distance and 75% of the distance?
2. Later, the class passed this sign. About what percent of the total distance have they travelled at this point?
3. Why was it helpful to know that 25% is $\frac{1}{4}$ and 50% is $\frac{1}{2}$ to solve the problem?

Checking

4. The Grade 8 students went on a 40 km canoe trip.



   a) The number line for the trip shows 10% of 40 km as 4 km. Explain how to determine 10% when you know 100%.
   b) Explain how to use 10% to determine 40%.
   c) Complete the number line for the trip.

Practising

5. Estimate or calculate. Show your work.
   a) 50% of the students in a class of 24 students
   b) 10% of the cost of a T-shirt if the whole cost is $12.99
   c) 25% of a 10 kg bag of sugar
6. Estimate the percent of each number. Show your work.
   a) 40% of 55 b) 5% of 160 c) 75% of 128
7. Estimate each. Show your work.
   a) 10% of a busload of 62 students
   b) 50% off shoes whose full price is $79.95
   c) 15% tip on a meal that cost $24.76

9 Unit Rates

You will need
- calculator
- counters
- play money

Goal **Represent relationships using unit rates.**

Each game at the class fun fair has a different cost.

? Which game is the best bargain?

Tom's Solution

Make a Plan

I can't compare the costs of each game the way they are written, but I could compare them if I knew the cost for one play in each game.

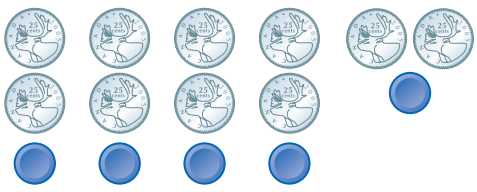
I will calculate the **unit rate** for plays in each game.

I will use counters. Each counter will represent one play.

Carry Out the Plan

For Sponge Toss, the rate is $2.50 for 5 plays.

If I use 8 quarters for $2.00, then I show 10 quarters for 5 plays.



This is the same as 2 quarters for each play, or 50¢.
The unit rate for the Sponge Toss is 50¢ for 1 play.

unit rate
A comparison of two quantities where the second one is described as 1 unit.

For example, a unit rate might be 30 km in 1 h or 4 tomatoes for $1

Communication Tip
Rates often have words like "per" or "for" in them. A slash (/) is sometimes used instead.

For example, you read 100 km/h as "100 km per hour."

A. Calculate the unit rate for one play of Super Darts and one play of Ball Toss.

B. Order the unit rates for one play from least to greatest.

C. Which game is the best bargain?

Reflecting

1. What does the word *unit* mean in the phrase *unit rate*?
2. Could Tom also have compared the prices by determining the number of plays for $1 in each game? Which way do you think is easier? Explain.
3. How is a unit rate like a ratio? How is it different?

Checking

4. Adrian wants to play the game that is the best bargain.
   a) Determine the number of plays for $1 in each game.
   b) Order the games from least unit rate to greatest unit rate.

Practising

5. Calculate the unit rate for each item.
   a) 6 batteries for $3.00
   b) 3 batteries for $1.95
   c) 2 CDs for $9.00
   d) 4 CDs for $16.00

6. Isaac found these heart rates of different animals.
   a) What is the unit heart rate in beats per minute for each animal?
   b) Order the unit rates from least to greatest.

Mammal	Heart Rate
rabbit	400 beats in 2 min
lion	40 beats in 1 min
shrew	400 beats in 30 s
elephant	140 beats in 4 min

7. a) What is the price for one of each type of cookie?
   b) Why is your answer to Part a) a unit rate?
   c) Use your unit rates to decide which cookie is the least expensive.

raisin cookies	5 cookies for $3.75
oatmeal cookies	7 cookies for $6.30
almond cookies	$4.00 for 8 cookies
carob chip cookies	$6.00 for 6 cookies

8. Who moved the fastest? Order these speeds from least to greatest.

Alexa walked 12 km in 3 h.
Alex cycled 10 km in 1 h.
Jenna cycled 8 km in 30 min.
Luke walked 10 km in 2 h.

9. A 200 g bag of chips costs $3.00 and a 300 g bag costs $4.00. Which bag is a better bargain? How do you know?

CHAPTER 12

10 Solving Problems Using Guess and Test

Goal **Use a guess and test strategy to solve problems.**

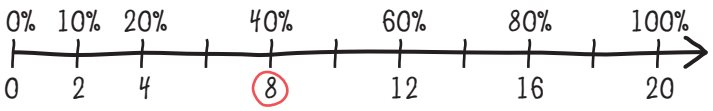
Cars make up 40% of Emilio's collection of 20 miniature cars and trucks. Emilio adds some cars to the collection and says, "Now I have 70% cars."

? How many cars did Emilio add to the collection?

Denise's Solution

Understand

Emilio started with 20 vehicles that made 100% of his collection.



If 100% is 20 cars, then 10% is 2 cars and 40% is $4 \times 2 = 8$ cars.

20 vehicles $-$ 8 cars $=$ 12 trucks.

When Emilio started, the ratio of cars to trucks was 8 : 12.

Make a Plan

I will set up a table and use a guess and test strategy to determine the number of cars he added.

Carry Out the Plan

Guess	Number of cars	Number of trucks	Number of cars and trucks	Percent of cars
Original	8	12	20	$\frac{8}{20} = 40\%$
Add 10 cars	$8 + 10 = 18$	12	30	$\frac{18}{30} = \frac{6}{10} = 60\%$
Add 20 cars	$8 + 20 = 28$	12	40	$\frac{28}{40} = \frac{7}{10} = 70\%$

When I tested adding 20 cars, the percent of cars was 70%.
Emilio added 20 cars to his collection.

Reflecting

1. How did the table help Denise solve this problem?
2. How did Denise test each of her guesses?

Checking

3. Engines make up 25% of Denise's collection of 12 miniature train engines and boxcars. After Denise added new engines, her collection was 50% engines. How many engines did she add?

Practising

4. The ratio of red to blue in a purple dye is 2 : 3. How many parts of blue need to be added to change the dye to 80% blue?
5. Lily grows beans in different soil mixtures. She starts with a 3 : 4 ratio of peat moss to sand. She wants a mixture that is 60% peat moss. How many parts of peat moss does she need to add?
6. a) Identify a pair of two-digit numbers whose product contains one 9.
   b) Identify a pair of two-digit numbers whose product contains two 9s.

Math Game

Ratio Concentration

You will need

- Ratio Concentration Cards

Number of players: 2 or more

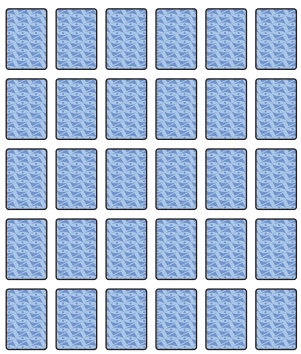
How to play: Match pairs of cards.

Assume the ratios are part to whole ratios.

Step 1 Shuffle the cards and place them face down in a 5 by 6 array.

Step 2 Take turns turning over two cards at a time.

Step 3 If the cards show equivalent ratios, keep them and take another turn. If the ratios are not equivalent, turn them face down again.



Step 4 Continue taking turns until all matches have been made.

The player with the greatest number of cards at the end wins.

Qui's Turn

I turn over these cards. The ratios are equivalent, so I keep the cards.

2:4	50%

Next I turn over these cards. The ratios are not equivalent. My turn is over.

$\frac{3}{5}$	30%

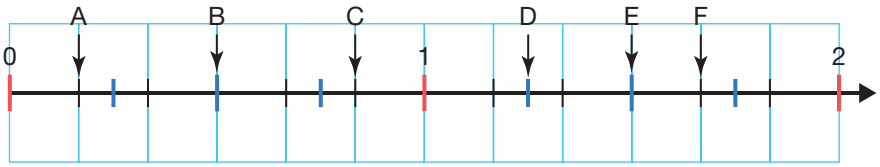
Skills Bank

LESSON 1

1. Name the letter on the number line that corresponds to each number.

a) $1\frac{1}{2}$ **c)** $1\frac{2}{3}$ **e)** $\frac{5}{4}$

b) $\frac{5}{6}$ **d)** $\frac{1}{6}$ **f)** $\frac{2}{4}$



LESSON 2

2. Compare. Write $>$, $<$, or $=$. Show your work.

a) $\frac{4}{5} \blacksquare \frac{3}{5}$ **c)** $\frac{1}{3} \blacksquare \frac{1}{5}$ **e)** $1\frac{3}{4} \blacksquare 1\frac{1}{2}$

b) $\frac{8}{3} \blacksquare \frac{5}{3}$ **d)** $\frac{7}{5} \blacksquare \frac{7}{4}$ **f)** $2\frac{1}{2} \blacksquare 2\frac{4}{8}$

3. Order each set of numbers from least to greatest.

a) $2\frac{1}{3}, \frac{5}{3}, \frac{2}{3}$

b) $1\frac{3}{8}, 2\frac{1}{8}, 1\frac{2}{4}$

c) $\frac{5}{3}, 1\frac{5}{6}, \frac{2}{3}, 1\frac{1}{3}$

LESSON 3

4. Write a decimal equivalent for each fraction.

a) $\frac{2}{5}$ **c)** $\frac{6}{5}$ **e)** $1\frac{6}{8}$

b) $\frac{1}{4}$ **d)** $\frac{7}{2}$ **f)** $2\frac{3}{4}$

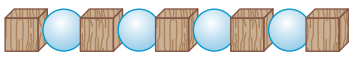
5. Write an equivalent fraction for each decimal.

a) 0.7 **c)** 0.47 **e)** 1.5

b) 0.30 **d)** 1.4 **f)** 2.62

LESSON 4

6. **a)** Write the ratio of glass beads to wooden beads.
b) Write the ratio of wooden beads to glass beads.



7. Leah makes rock candy with 1 cup of water and 5 cups of sugar.

a) Model the ratio of sugar to water with counters.

b) Write the ratio of sugar to water.

4

8. Chase made Aboriginal soft-bread from a recipe that read "Mix the cups of white corn flour to sugar in a 5 to 1 ratio."

a) If Chase used 10 cups of white corn flour, how many cups of sugar did he use?

b) Write another part-to-part ratio for the recipe.

c) Write four other ratios describing the flour and sugar mixture.

5

9. Copy and complete each equivalent ratio.

a) 1 to 8 = ■ to 64

b) 4 to 10 = ■ to 70

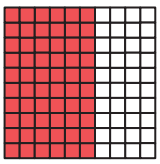
c) 12 to14 = ■ to 7

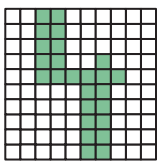
d) 18 to 2 = ■ to 1

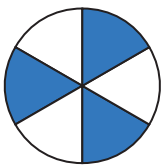
10. Sabrina wants to make a carving from plaster of Paris. She will make the plaster of Paris using a ratio of 2 parts plaster to 1 part water. If she uses 6 cups of plaster of Paris, how many cups of water will she need to add?

6

11. What percent of each whole is coloured?

a) 

b) 

c) 

12. Write each ratio as a percent. Use a hundredths grid.

a) 92 to 100

b) 12 to 20

c) $\frac{9}{25}$

d) $\frac{8}{40}$

e) 8 : 100

f) 40 : 50

13. What percent might you use to describe each?

a) all

b) almost all

c) none

d) almost none

e) a little over half

f) just less than one fourth

7

14. Write each set of numbers as percents and order them from least to greatest.

a) 16%, $\frac{1}{10}$, 0.14, 0.9

b) 0.98, $\frac{12}{15}$, 87%, 0.89

c) 40%, 0.59, $\frac{6}{8}$, 0.38

7 **15.** Data was collected on how people in Toronto travel to work.

a) Copy and complete the table.

b) Order the methods of travel from least to greatest.

Method of travel	Fraction	Decimal	Percent
Car			72%
Walk/bicycle	$\frac{3}{50}$		
Public transit		0.22	

8 **16.** Calculate 10% and 25% of each measurement. Show your work.

a) a 100 m swimming pool

b) a 10 km run

c) a 60 min class

d) a group of 440 people

17. Estimate or calculate each percent. Show your work.

a) 75% of 40

b) 50% of 212

c) 10% of $189

d) 40% of $50

e) 15% of 60 km

f) 20% of 67 people

9 **18.** For each book, about how many pages are in one chapter?

a) a three-chapter book with 45 pages

b) a five-chapter book with 70 pages

c) a six-chapter book with 90 pages

19. Four families are travelling to Niagara Falls from different places. Order their speeds from least to greatest.

Sapons	80 km in 2 h
Silvers	180 km in 3 h
Johns	50 km in 1 h
Cunninghams	35 km in 30 min

20. Which store has the best bargain for kiwis?

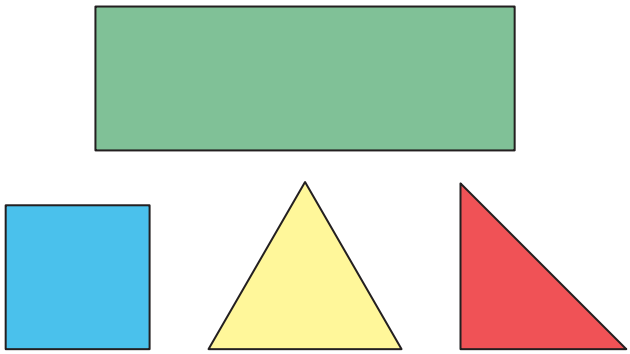
Ken's Grocery	2 kiwis for 95¢
Davis Bay Grocery	4 kiwis for $1.80
ML Variety Store	3 kiwis for $1.50

Problem Bank

LESSON

2 **1.** Jeff is $\frac{2}{3}$ of a year older than Lisa. Lisa is $\frac{1}{4}$ of a year older than Mark. If Mark is $10\frac{1}{2}$, how old is Lisa?

5 **2.** The ratio of the length of one side of a shape to its perimeter is 1 : 3. Could the shape be a triangle? a square? a rectangle? Explain your thinking.



3. What are two ratios equivalent to 1 million : 1 billion?

4. A recipe for fruit punch calls for a juice to soda water ratio of 3 : 2. The ratio of fruit juices is 1 : 3 : 2 for grape to apple to raspberry. If Diana starts with 4 L of raspberry juice, how many litres of grape juice, apple juice, and soda water should she add?

6 **5.** Eight of the 20 players forming two indoor soccer teams are experienced. Once the teams are formed, team A has 60% experienced players.

a) What is the ratio of experienced to inexperienced players on team A? Show your work.

b) What is the percent of inexperienced players on team B? Show your work.

8 **6.** Carolyn took $64.00 spending money on a trip. At the end of the first day she had 75% of the money left. Calculate the amount of money she had spent.

9 **7.** Ivan can type 48 words each minute. About how long would it take him to type this page of the text? Explain how you calculated.

Frequently Asked Questions

Q: What is percent?

A: Percent is a special part to whole ratio where the whole is 100.

For example, 5 students in a class say they have dogs as pets. There are 20 students in the class. The ratio of dog owners to students in the class is 5 to 20. This is a part to whole ratio.

To write this as a percent, write an equivalent ratio with 100 as the whole. 5 to 20 = 25 to 100.

Q: How do you compare percents, fractions, and decimals?

A: It is easiest to compare fractions, percents, and decimals when they are all represented as part of the same whole and written the same way: all as fractions, all as decimals, or all as percents.

If the values are fractions, you can write them as equivalent fractions with denominators 100. Then they can be compared as percents or decimal hundredths.

For example: Compare $\frac{6}{8}$, 68% and 0.7.

$\frac{6}{8} = \frac{3}{4} = \frac{75}{100} = 75\%$

$0.7 = \frac{7}{10} = \frac{70}{100} = 70\%$

From least to greatest, these numbers are 68%, 0.7 or 70%, and $\frac{6}{8}$ or 75%.

Q: What is a unit rate?

A: A unit rate is a way to compare quantities described in different ways.

For example, different sizes of a product will be priced differently. Determining unit rates for each size is a way to decide which size is the best bargain.

You can determine which size gives you the most product for $1, or which product costs the least for 1 unit.

LESSON

Chapter Review

1

1. As an Earth Day project, five friends started compost bins for their families. At the end of the summer, they measured how much compost they had. Order the numbers from least to greatest. Who had the most compost?

Friend	Number of bins
Jake	$\frac{2}{5}$
Kaycee	$1\frac{1}{2}$
Julie	$\frac{4}{5}$
Kim	$1\frac{1}{5}$
Gab	$\frac{7}{2}$

3

2. Write each fraction as a decimal equivalent.

a) $\frac{4}{5}$ **b)** $\frac{3}{2}$ **c)** $1\frac{7}{8}$ **d)** $1\frac{2}{5}$

3. Write each decimal as an equivalent fraction

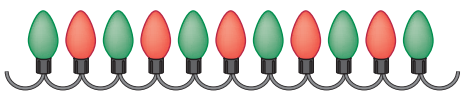
a) 0.9 **b)** 0.75 **c)** 1.4 **d)** 2.35

4

4. Write the ratio of red to green in each set.

a)

b)



5. An orchard has 6 pear trees and 5 apple trees. Write six ratios about the trees in the orchard.

5

6. Paris makes trail mix with 5 cups nuts to 2 cups pretzels.

a) Write the ratio of nuts to pretzels.

b) Use the ratio to complete the table of values.

c) If Paris has 18 cups of pretzels, how many cups of nuts does she need?

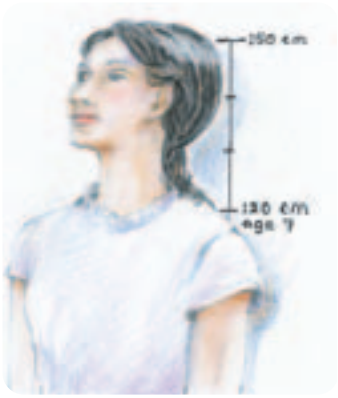
Nuts	10				25	15
Pretzels		8	2	12		

6

7. A garden of white and red roses has 60% red roses. What percent of the roses are white?

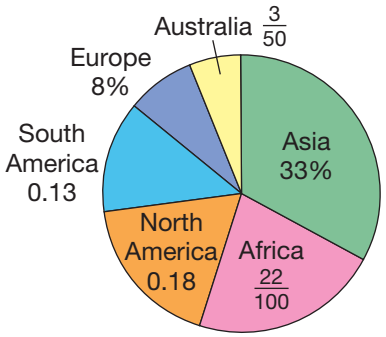
8. **a)** What is the ratio of Jennifer's height now to her height at age 7?

b) What percent of Jennifer's present height is her height at age 7?



7

9. Alana used various sources to describe the ratio of land area on each continent to the total area of land on Earth (not including Antarctica). She used various sources.



a) Copy and complete the table.

Continent	Fraction	Decimal	Percent
Asia			
Africa			

b) Order the continents from least to greatest area of land.

8

10. A group of 80 people went on a trip. Twenty-five percent of the travellers had 2 bags each. The rest had 1 bag. How many bags were there altogether?

9

11. Cole researched the average speeds of some animals. Which animal is the slowest?

Animal	Speed
snail	6 cm in 4 s
spider	38 cm in 2 s
giant tortoise	45 cm in 6 s
slug	4 cm in 8 s
centipede	100 cm in 2 s

10

12. A chemist made a mixture of 3 parts copper to 5 parts gold. He wants to change the mixture so that it is 85% gold. How many parts of gold must he add?

Chapter Task

Running with Terry

Terry Fox began his run across Canada in St John's, Newfoundland in April, 1980. The map shows his route across the island.

Scale: 1 cm represents 50 km

? What fraction and percent of Terry's run was completed in Newfoundland?

Part 1: Newfoundland

A. Use string to measure the length of the line representing Terry's route through Newfoundland. Express your answer in centimetres.

B. The scale on the map shows that 1 cm on the map represents 50 km. What is the length of Terry's route in kilometres?

C. Terry ran about 35 km each day. About how many days did he take to get across Newfoundland?

Part 2: The Whole Trip

Terry ran about 5373 km in 143 days before his cancer forced him to stop in Thunder Bay, Ontario.

D. Draw a percent line from 0% to 100% showing the total time and total distance he ran.

E. Locate the times and distances for the part of the run through Newfoundland on the percent line and determine the percent of the run completed there.

F. Estimate the fraction of the total distance and the fraction of the time he spent in Newfoundland.

Task Checklist

- ☑ Did you show your calculations?
- ☑ Did you include units on your percent line?
- ☑ Did you label your percent line with benchmark percents?
- ☑ Did you check your work?