You will be able to
• compare and order decimals and percents
• relate fractions, ratios, decimals, and percents
• solve problems that involve ratios, rates, and percents
• multiply and divide decimal numbers
Making Number Comparisons

In the fall, some Canadian farmers build mazes in their cornfields to attract tourists and earn extra money. The grid shows a plan for a simple maze in a square cornfield. The ratio of green squares (with corn) to white squares (paths without corn) is 58:42.

? How can you use ratios, fractions, decimals, and percents to describe a path through the maze?

A. What fraction of the square cornfield is used for paths in the maze? Write the decimal that is equivalent to this fraction.

B. What percent of the cornfield is used for paths in the maze?

C. What fraction of the cornfield is used for corn? Write the decimal that is equivalent to this fraction.

D. What percent of the cornfield is used for corn?

E. Copy the maze and find a path from Start to Finish.

F. Count the white squares on your path. Use ratios, fractions, decimals, and percents to express the area of your path as part of the total area of the cornfield.

G. Explain why the areas of the paths are easily expressed as ratios, fractions, decimals, and percents of the cornfield’s total area.
Do You Remember?

1. The number 100 has factors 2 and 50, since \(2 \times 50 = 100\). List all the other factor pairs of 100.

2. Yan buys a candy bracelet that has 24 candy beads. Write each ratio, based on the picture of the bracelet.
   a) the number of red beads to the number of blue beads
   b) the number of green beads to the number of red beads
   c) the number of yellow beads to the number of white beads
   d) the number of green beads to the total number of beads
   e) the total number of beads to the number of green beads

3. Which ratios are equivalent to 4:5?
   a) 16:20  c) 9:10
   b) 12:15  d) 90:100  e) 32:40

4. Express each shaded area as a fraction of the area of the whole shape.
   a) \[
   \begin{array}{cccccc}
   \text{Shaded} & \text{Unshaded} \\
   \text{Row 1} & \text{Row 2} & \text{Row 3} & \text{Row 4} & \text{Row 5} \\
   \text{Cell 1} & \text{Cell 2} & \text{Cell 3} & \text{Cell 4} & \text{Cell 5} \\
   \hline
   \text{Shaded} & \text{Unshaded} \\
   \text{Row 1} & \text{Row 2} & \text{Row 3} & \text{Row 4} & \text{Row 5} \\
   \text{Cell 1} & \text{Cell 2} & \text{Cell 3} & \text{Cell 4} & \text{Cell 5} \\
   \end{array}
   \]
   b) \[
   \begin{array}{cccc}
   \text{Shaded} & \text{Unshaded} \\
   \text{Row 1} & \text{Row 2} \\
   \text{Cell 1} & \text{Cell 2} \\
   \hline
   \text{Shaded} & \text{Unshaded} \\
   \text{Row 1} & \text{Row 2} \\
   \text{Cell 1} & \text{Cell 2} \\
   \end{array}
   \]

5. Copy the following number line, and mark each number on your number line.

6. Look at the numbers you marked on your number line in question 5. Which numbers are equivalent?

7. For each shape, write the following.
   a) the ratio that compares the number of shaded squares to the number of unshaded squares
   b) the fraction that expresses the area of the shaded squares as part of the area of the whole shape
   c) the decimal and percent that are equivalent to this fraction
   i) \[
   \begin{array}{cccccc}
   \text{Shaded} & \text{Unshaded} \\
   \text{Row 1} & \text{Row 2} & \text{Row 3} & \text{Row 4} & \text{Row 5} \\
   \text{Cell 1} & \text{Cell 2} & \text{Cell 3} & \text{Cell 4} & \text{Cell 5} \\
   \hline
   \text{Shaded} & \text{Unshaded} \\
   \text{Row 1} & \text{Row 2} & \text{Row 3} & \text{Row 4} & \text{Row 5} \\
   \text{Cell 1} & \text{Cell 2} & \text{Cell 3} & \text{Cell 4} & \text{Cell 5} \\
   \end{array}
   \]
   ii) \[
   \begin{array}{cccc}
   \text{Shaded} & \text{Unshaded} \\
   \text{Row 1} & \text{Row 2} \\
   \text{Cell 1} & \text{Cell 2} \\
   \hline
   \text{Shaded} & \text{Unshaded} \\
   \text{Row 1} & \text{Row 2} \\
   \text{Cell 1} & \text{Cell 2} \\
   \end{array}
   \]

8. Use mental math to multiply.
   a) \(235 \times 0.1\)  c) \(876 \times 0.01\)
   b) \(235 \times 0.01\)  d) \(876 \times 0.001\)

9. Multiply. Check by using a calculator.
   a) \(9 \times 11\)  c) \(4 \times 52\)
   b) \(9 \times 0.11\)  d) \(4 \times 5.2\)

10. Guess and test to determine the missing number.
    a) \(2 \times \_ = 4\)  d) \(100 \times \_ = 160\)
    b) \(2 \times \_ = 3\)  e) \(473 \times \_ = 4.73\)
    c) \(20 \times \_ = 30\)  f) \(111 \times \_ = 1.11\)

11. Divide. Check by using a calculator.
    a) \(67 \div 10 = \_\)  d) \(67 \div \_ = 6.7\)
    b) \(670 \div 100 = \_\)  e) \(200 \div \_ = 2.00\)
    c) \(30 \div \_ = 3.0\)  f) \(473 \div \_ = 4.73\)

12. Divide. Check by using a calculator.
    a) \(12 \div 12 = \_\)  c) \(5 \div \_ = 1\)
    b) \(2.5 \div 2.5 = \_\)  d) \(5.5 \div \_ = 1\)
2.1 Exploring Ratio Relationships

**GOAL**
Explore equivalent ratios.

**Explore the Math**

Fawn drew a red rectangle like this one on centimetre grid paper. She wants to create a series of similar rectangles.

![Diagram of a red rectangle on a grid]

- **How do you draw similar rectangles?**

  **A.** Use centimetre grid paper to draw a rectangle exactly like Fawn’s rectangle. Use a ruler to draw a diagonal from the bottom left corner to the top right corner of your rectangle. Extend the diagonal as far as you can on the grid paper.

  **B.** Create a new rectangle by adding 2 cm to the width and 3 cm to the length of your original rectangle. Start this rectangle at the bottom left of the first rectangle so that the two rectangles overlap.

  **C.** Repeat step B to create four more rectangles. Each time add 2 cm to the width and 3 cm to the length of the previous rectangle. You will have six rectangles in total.

**similar rectangles**
rectangles that have the same shape, but not necessarily the same size
D. Copy the following table. Use your rectangles to complete the table.

<table>
<thead>
<tr>
<th>Width (cm)</th>
<th>First Group of Rectangles</th>
<th>Length (cm)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Width:Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

E. On a new piece of grid paper, draw another rectangle that has a width of 2 cm and a length of 3 cm. Use a ruler to draw a diagonal from the bottom left corner to the top right corner of this rectangle. Extend the diagonal as you did in step A. Create five new rectangles by adding 3 cm to the width and 3 cm to the length each time.

F. Copy the following table. Use your rectangles from step E to complete the table.

<table>
<thead>
<tr>
<th>Width (cm)</th>
<th>Second Group of Rectangles</th>
<th>Length (cm)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Width:Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

G. Choose a length and a width for a rectangle. Draw the rectangle on grid paper. Draw and extend the diagonal as in step A. Draw five rectangles that are similar to your rectangle.

Reflecting

1. Which groups of rectangles contain similar rectangles? How do you know?

2. In which groups of rectangles are the ratios of width to length equivalent? What is the connection between ratios of sides and whether rectangles are similar?

3. In step E, did the diagonal pass through each rectangle’s top right corner? Explain why or why not.

4. If you start with any rectangle, how can you create a rectangle that is similar to it?
Snowboarding is the fastest growing winter sport in Canada. A recent estimate of the Canadian male-to-female participant ratio is 3:1. Romona’s school is going on a ski trip, and 24 boys signed up to go snowboarding.

How many girls signed up to go snowboarding if the numbers match the Canadian ratio?

You can write equivalent ratios by multiplying or dividing each term in a ratio by the same number. For example,

\[
\begin{align*}
\times 2 & \quad \frac{1:3}{2:6} \\
\frac{1}{3} & = \frac{2}{6} \\
\times 2 & \\
\div 4 & \quad \frac{16:12}{4:3} \\
\frac{16}{12} & = \frac{4}{3} \\
\div 4 & \\
\end{align*}
\]

From this ratio, I can see that 8 girls signed up to go snowboarding.

The ratio of male to female snowboarders is 3:1 or \(\frac{3}{1}\). First I multiplied each term by 5. Then I continued to write ratios that are equivalent to \(\frac{3}{1}\) until I found the ratio with the term for 24 boys.
Example 2: Calculating the missing term

Calculate the number of girls who signed up to go snowboarding.

Paul’s Solution

First I wrote the Canadian ratio of boys to girls, which is 3:1. I want to find out the number of girls when there are 24 boys, so I wrote the second ratio as 24: ■. The ratios must be equivalent. Since $3 \times 8 = 24$, I multiplied by 8 to get the missing term in the second ratio.

The number of girls who signed up to go snowboarding must be $1 \times 8 = 8$.

Communication Tip

Ratios can compare the pieces in this diagram in different ways: part to whole, whole to part, or part to part.

• part to whole
  The number of red pieces to the total number of pieces is 3 to 8, $\frac{3}{8}$, or 3:8.
  The number of blue pieces to the total number of pieces is 5 to 8, $\frac{5}{8}$, or 5:8.

• whole to part
  The total number of pieces to the number of red pieces is 8 to 3, $\frac{8}{3}$, or 8:3.
  The total number of pieces to the number of blue pieces is 8 to 5, $\frac{8}{5}$, or 8:5.

• part to part
  The number of red pieces to the number of blue pieces is 3 to 5, $\frac{3}{5}$, or 3:5.
  The number of blue pieces to the number of red pieces is 5 to 3, $\frac{5}{3}$, or 5:3.

Reflecting

1. How did Paul decide what scale factor to use to solve his proportion?

2. a) How can you create a ratio that is equivalent to another ratio?
   b) How can you determine whether two given ratios are equivalent?

3. Suppose that you have a proportion in which one term is missing from one of the ratios. How can you determine the value of the missing term?

4. a) How are ratios the same as fractions?
   b) How are ratios different from fractions?
Example 3: Dividing and multiplying to write equivalent ratios

Determine the missing term in each proportion.

a) \(18:9 = 8:\square\)

b) \(\frac{15}{25} = \frac{18}{\square}\)

Solution

Divide the terms in the first ratio to write a simpler ratio. Then multiply to calculate the value of the missing term.

\[\begin{align*}
\text{a) } & \quad 18:9 = 2:1 \quad \text{and} \quad 2:1 = 8:4 \\
\text{b) } & \quad \frac{15}{25} = \frac{3}{5} \quad \text{and} \quad \frac{3}{5} = \frac{18}{30}
\end{align*}\]

The missing term is 4. The missing term is 30.

A Checking

5. Use the ratios represented in each diagram.
   a) Write the two ratios as a proportion.
   b) Determine the scale factor that relates the two ratios.
   c) Calculate the missing term.

i) ii) iii) iv)

6. Calculate each missing term.
   a) \(\frac{3}{8} = \square\)
   b) \(\frac{32}{24} = \frac{20}{\square}\)

B Practising

7. Copy and shade the grid on the right so that the ratio of shaded squares to total squares is the same as the ratio in the grid on the left. Then write the proportion.

a) 
   b) 

8. Write three equivalent ratios for each of the following ratios.
   a) 21 to 56
   b) 6:54
   c) 48:36
   d) \(\frac{22}{55}\)
9. The ratio of the number of orange sections to the total number of sections is the same for all three diagrams. Explain why.

10. Determine the missing term in each proportion.
   a) \( \frac{27}{45} = \frac{x}{5} \)
   b) \( \frac{2}{6} = \frac{3}{x} \)

11. Measure the length or height of the animal in each scale drawing. Use your measurement and the given scale to calculate the actual length or height of each animal.
   a) scale 1:20
   b) scale 1:100
   c) scale 2:1

12. The height of the CN Tower is about 550 m. What is the height of the CN Tower in a scale drawing if the scale is 1:1 1000?

13. Write each comparison as a ratio. The units for the terms must be the same.
   a) 400 g to 1 kg
   b) 6 cm to 7 mm
   c) 200 s to 3 min

14. Katherine spends 7 h a day in school. This includes a 30 min lunch break each day.
   a) Write a ratio that compares the time for her lunch break with the total time she spends at school each day.
   b) Write a ratio that compares the time for her lunch breaks for a week with the total time she spends at school each week.
   c) Determine the number of hours of lunch break she has in a month of school days.

15. For the numbers 2 to 100, including 2 and 100, determine each ratio.
   a) the number of even numbers to the number of odd numbers
   b) the number of multiples of 6 to the number of multiples of 8
   c) the number of prime numbers to the number of composite numbers

16. Todd knows that the ratio of boys to girls in his class is 3:5. Since 12 of the students are boys, he says there must be 36 students in his class. Is he right? Explain your thinking.

17. Luca is 9 years old and 129 cm tall. Medical charts show that a boy’s height at age 9 is \( \frac{3}{4} \) of his predicted adult height. Predict Luca’s adult height.

18. At Darlene’s Dairy Bar, the ratio of vanilla to chocolate to strawberry ice cream cones sold is about 3:5:2.
   a) Last week, 155 chocolate ice cream cones were sold. What was the total number of ice cream cones sold?
   b) Two weeks earlier, a total of 380 ice cream cones were sold. How many of each flavour were sold?
Peter has started a new part-time job making pizza at Pizza Shack. For every 3 h he works, he earns $27.00.

How can you calculate Peter’s earnings if he works 24 h in a week?

A. Write the rate for Peter’s earnings in 3 h, using 3 h as the second term.
B. Write a rate with a missing term for Peter’s earnings in 24 h.
C. The rates in steps A and B are equivalent rates. Write a proportion and solve it.

Communication Tip

Rates are often written with a slash (/). For example, the maximum speed on a highway is 100 km/h. The slash is read as “per,” meaning “for each” or “for every.” The rate of 100 km/h means that, on average, you are travelling a distance of 100 km for every hour you are driving.

Reflecting

1. a) Explain why the rates in steps A and B are equivalent.
   b) Explain how you know that Peter’s hourly rate of pay is $9/h.
2. Why is it important to include the units in a rate?
3. a) How are a rate and a ratio the same?
   b) How are they different?
4. How is solving a rate problem like solving a ratio problem?
Work with the Math

Example 1: Calculating a rate

On a recent trip from Kingston to Toronto, Brooke's uncle drove the 240 km in 3 h.

a) What was the average rate that he drove?

b) Suppose that he took 2.5 h to drive the distance. What would the average rate be?

Romona’s Solution

\[ \frac{240 \text{ km}}{3 \text{ h}} = \frac{\text{ km}}{1 \text{ h}} \]

I compared the distance to the time using a rate. I wrote a proportion with a missing term for the distance travelled in 1 h. The scale factor is 3 because \( 3 \div 3 = 1 \). So, I divided 240 by 3 to get the missing term.

Brooke’s uncle drove at an average rate of 80 km/h.

\[ \frac{240 \text{ km}}{2.5 \text{ h}} = \frac{\text{ km}}{1 \text{ h}} \]

I wrote a proportion with a missing term for the distance travelled in 1 h. The scale factor is 2.5 because \( 2.5 \div 2.5 = 1 \). I divided 240 by 2.5 to get the missing term.

Brooke’s uncle drove at an average rate of 96 km/h.

Example 2: Solving a proportion

Jean-Pierre earns $6.00 for every 50 newspapers that he delivers. If Jean-Pierre delivers 450 newspapers in a week, how much will he earn?

Miguel’s Solution: Finding a scale factor

\[ \frac{50 \text{ papers}}{\$6} = \frac{450 \text{ papers}}{\text{earnings in dollars}} \]

I used “earnings in dollars” to represent Jean-Pierre’s earnings for the week. I wrote a proportion. I decided to use the number of papers delivered as the top term and earnings as the bottom term.

\[ 50 \text{ papers} \times \_ = 450 \text{ papers} \]

I have to find a number that multiplies by 50 to give me 450. I know that \( 450 \div 50 = 9 \), so \( 50 \times 9 = 450 \).

\[ \frac{50 \text{ papers} \times 9}{\$6 \times 9} = \frac{450 \text{ papers}}{\$54} \]

I calculated his earnings by multiplying \( 6 \times 9 = 54 \).

Jean-Pierre will earn $54 if he delivers 450 papers.

Bonnie’s Solution: Using logical reasoning

\[ \frac{\$6 \times 2}{50 \text{ papers} \times 2} = \frac{\$12}{100 \text{ papers}} \]

I know that Jean-Pierre gets $6 for delivering 50 papers, so he will get twice that for 100 papers. That’s $12.

\[ \frac{\$12 \times 4}{100 \text{ papers} \times 4} = \frac{\$48}{400 \text{ papers}} \]

So, he will get 4 times that for 400 papers. That’s $48.

\[ \$48 + \$6 = \$54 \]

Altogether, he will get $48 + $6 for 450 papers.

Jean-Pierre will earn $54 for delivering 450 papers.
Checking

5. Write each comparison as a rate.
   a) There was 15 mm of rain over 3 days.
   b) Four chocolate bars were on sale for $2.20.
   c) Indu saves $14.00 every week.
   d) Philip’s height changed by 12 cm over 4 months.

6. Write two equivalent rates for each comparison.
   a) 5 goals in 10 games
   b) 10 km jogged in 60 min
   c) 6 pizzas eaten in 30 min
   d) 10 penalties in 25 games

Practising

7. On a hike, Peter walked 28 km in 7 h.
   a) What was his average rate of walking?
   b) Suppose that he walked the 28 km in 8 h. What would his average rate of walking be?

8. Write a proportion for each situation. Determine the missing term in each proportion.
   a) Three trucks have 54 wheels. Six trucks have □ wheels.
   b) In 5 h, you drive 400 km. In 1 h, you can drive □ km.
   c) In 1 h, you earn $10. In 8 h, you can earn $□.
   d) Six boxes contain 72 donuts. One box contains □ donuts.
   e) In 4 min, you can type 128 words. In 2 min, you can type □ words.
   f) Six pencils that cost $0.72 is equivalent to three pencils that cost □.

9. Winnie’s baseball coach orders 4 pizzas for the 10 players on her team. She needs to determine the number of pizzas to order for the league’s year-end party that 120 players are expected to attend. Which proportion does not model this situation?

   A. \[
   \frac{4 \text{ pizzas}}{10 \text{ players}} = \frac{□ \text{ pizzas}}{120 \text{ players}}
   \]
   B. \[
   \frac{4 \text{ pizzas}}{10 \text{ players}} = \frac{120 \text{ players}}{□ \text{ pizzas}}
   \]
   C. \[
   \frac{10 \text{ players}}{4 \text{ pizzas}} = \frac{120 \text{ players}}{□ \text{ pizzas}}
   \]
   D. \[
   \frac{2 \text{ pizzas}}{5 \text{ players}} = \frac{□ \text{ pizzas}}{120 \text{ players}}
   \]

10. Takumi bikes 45 km in 3 h. If he continues at the same rate, how far will he bike in 4 h?

11. Brad pays $56 for four CDs. At this rate, how many CDs can he buy with $42?

12. Tony works at a gas station. He serves 12 cars in 20 min. At this rate, how many cars can he serve in 25 min?

13. Anita earns $72 every 6 h to fix bicycles. How much money will she earn in 4 h if she is paid at the same rate?
14. Craig jogs 12 km in 1 h. If he jogs at the same rate, how long will it take him to jog 18 km?

15. If 6 kg of oranges costs $15, how many kilograms of oranges can you buy for $20?

16. The goalie for the Eagles stopped 8 shots out of every 10 shots.

19. A grey whale’s heart beats 24 times in 3 min. At this rate, how many times does it beat in a day?

17. Explain the difference between a ratio and a rate. Provide two examples in your explanation: a situation in which a ratio would be used and another situation in which a rate would be used.

18. Jason’s mom drove at 100 km/h for a 160 km trip. How much longer would the trip have taken if she had driven at 90 km/h?

20. Dan’s computer reads 68% of battery left after 1 h. If the rate of using power remains the same, about how much longer will his battery provide power?

21. Fuel efficiency is a rate that represents the number of litres of fuel required for the driving distance of 100 km. For example, the fuel efficiency for a certain model of car is \( \frac{9.5 \text{ L}}{100 \text{ km}} \). To compare the fuel efficiency of different cars, you need to determine the number of litres of fuel used per 100 km for each car.

<table>
<thead>
<tr>
<th>Model</th>
<th>Kilometres driven (km)</th>
<th>Litres of fuel used (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chevrolet Corvette</td>
<td>180</td>
<td>25.00</td>
</tr>
<tr>
<td>Honda Civic</td>
<td>275</td>
<td>22.36</td>
</tr>
<tr>
<td>Honda Accord</td>
<td>126</td>
<td>11.45</td>
</tr>
<tr>
<td>Ferrari F40</td>
<td>309</td>
<td>40.66</td>
</tr>
<tr>
<td>Ford Escort</td>
<td>294</td>
<td>25.56</td>
</tr>
<tr>
<td>Ford Taurus</td>
<td>258</td>
<td>27.74</td>
</tr>
<tr>
<td>Porsche 911 Turbo</td>
<td>333</td>
<td>46.25</td>
</tr>
</tbody>
</table>

a) Which car uses the least fuel? Explain.
b) Which car uses the most fuel? Explain.
Communicating about Ratio and Rate Problems

**GOAL**
Explain your thinking when solving ratio and rate problems.

**Communicate about the Math**

Alice works for a special effects company. She is building a scale model of the Lion’s Gate Bridge in Vancouver, British Columbia, to be used for a movie. The distance between the vertical supports on the bridge is 473 m, and the towers are 111 m tall. She decides to make the distance between the vertical supports on the model about 5 m. What should the height of the towers be on the model?

Tien’s Explanation

From the actual bridge, I know the ratio

\[ \frac{\text{tower height (m)}}{\text{distance between supports (m)}} = \frac{111}{473}. \]

For the model, I decided to make the distance between the supports 4.73 m.

For the model, the same ratio is

\[ \frac{\text{tower height (m)}}{\text{distance between supports (m)}} = \frac{111}{473}. \]

So \[ \frac{111}{473} = \frac{1.11}{4.73}. \]

The height of the towers of the model must be 1.11 m, or 111 cm.

**Nathan’s Questions**

Why did you write the ratio the way you did?

Why did you choose 4.73 m?

How did you calculate 1.11?

How do you know this is correct?

? How can Tien improve her solution to the problem?
A. Which of Nathan’s questions do you think are good questions? Why?

B. How should Tien respond to the questions?

C. What other questions do you think would be helpful?

D. Use the Communication Checklist to improve Tien’s solution.

Reflecting

1. Which parts of the Communication Checklist did Tien cover well? Explain.

2. Why should you explain your thinking when solving problems?

Work with the Math

Example: Communicating about ratios

The Coyotes have won 8 of their first 20 soccer games. If this continues, how many games would you expect them to win out of 30 games? Explain your thinking.

Fawn’s Solution

\[
\begin{align*}
\frac{8}{20} & = \frac{2}{5} \\
\frac{2 \times 6}{5 \times 6} & = \frac{12}{30} = \frac{2}{5}
\end{align*}
\]

Winning 8 out of 20 games is the same as winning 2 out of 5 games.

The team is playing 30 games. Since \(5 \times 6 = 30\), the team should win \(2 \times 6 = 12\) games out of 30 games.

The answer is correct because \(\frac{8}{20} = \frac{2}{5}\) and \(\frac{12}{30} = \frac{2}{5}\).

Also, you can use logical thinking to check the answer. If the team played half of 20, or 10, games, they would win half of 8, or 4, games. This means that they win 4 games out of every 10, or 12 games out of 30.

Miguel’s Solution

\[
\begin{align*}
\frac{8}{20} & = \frac{1}{3} \\
\frac{8}{20} & = \frac{12}{30} \\
\frac{8}{20} & = \frac{12}{30} \\
\times 1.5 \\
\times 1.5
\end{align*}
\]

I know that the Coyotes have won 8 games out of 20 games, and there are 30 games in total.

I wrote a proportion with a missing term for the number of games they will win.

The scale factor is 1.5 because \(20 \times 1.5 = 30\). So, I multiplied 8 by 1.5 to get the missing term.

The team should expect to win 12 games out of 30.

The answer is correct because the team has 10 more games to play, which is half of 20. They should win half of the number they’ve already won (8), which is 4.
Chapter 2

**Checking**

3. Marlene runs 4 km in 30 min. At this rate, can Marlene run 6 km in 45 min? Complete the following calculation and explanation. Use the Communication Checklist.

\[
\frac{4 \text{ km}}{30 \text{ min}} = \frac{\text{distance}}{\text{time}}
\]

I know that Marlene runs 4 km in 30 min. I can write a proportion to show this information.

7. A photocopier can make 1800 copies in 1 h. Rosa says that the photocopier can make 60 copies per minute. Is she correct? Explain why or why not.

8. For every 1.5 m that an iceberg rises above the water, there is 12 m of ice below the surface. If an iceberg rises 9 m above the water, is the height of the iceberg from top to bottom 72 m? Explain.

**Practising**

Use the Communication Checklist to help you answer each question.

4. Chris is having a party for 90 people. He has a punch recipe that makes 2 L of punch. This will serve 15 people. Chris thinks that he will need 12 L of punch for his party. Is Chris’s reasoning correct? Explain.

5. Sam has a part-time job delivering flyers door to door. He earns a quarter for every 10 flyers he delivers. He wants to earn $45.00 to buy his mother a birthday present.

   Sam figures out that he can write a proportion to determine how many flyers he needs to deliver. He thinks, “I know that I earn $0.25 for 10 flyers, and I want to know how many flyers I need to deliver to earn $45.00.”

   Complete Sam’s explanation, and solve the problem.

6. Akeem measured the capacity of a glass as 270 mL and the capacity of a Thermos® as 1 L. He said that the ratio 270 : 1 compares the capacity of the glass with the capacity of the Thermos®. Is his reasoning correct? Explain.

9. In a bulk-food store, peanuts in the shell are sold by their mass in grams. The rate is 44¢ per 100 g. In a grocery store, the price of a 400 g bag of peanuts in the shell is $2.20.

   Which would you buy to get the most peanuts for your money? Justify your choice.

10. These are the prices for two types of raisins in a bulk-food store.

   golden raisins …… $0.66/100 g
   dark raisins .......... $0.55/100 g

   a) A recipe calls for 500 g of raisins. How much will you save if you buy the less expensive raisins? Explain.
   b) Raj bought $3.63 worth of raisins. What was the mass in grams? Give two possible answers.
Did you know?
• It takes 21 kg of milk to produce 1 kg of butter.
• About 160 L of maple sap are needed to make 4 L of maple syrup.
• It takes 250 mL of uncooked rice to make 500 mL of cooked rice.
• It takes 2 kg of soybeans to make 5 kg of tofu.
• One cup of unpopped corn yields 36 cups of popped corn.

1. Write a proportion to describe each situation. Then answer the question.
   a) How much milk does it take to produce 25 kg of butter?
   b) How much maple sap is needed to make 60 L of maple syrup?
   c) How much uncooked rice is used to make 800 mL of cooked rice?
   d) What mass of soybeans is needed to make 3000 g of tofu?
   e) How much unpopped corn does it take to produce 72 cups of popped corn?

2. Write a proportion to describe each situation. Then answer the question.
   a) How much butter can be made with 6300 kg of milk?
   b) How much maple syrup can be made with 80 L of maple sap?
   c) How much cooked rice can be made with 1 L of uncooked rice?
   d) What mass of tofu can be made with 8000 g of soybeans?
   e) How much popped corn can be made with 8 cups of unpopped corn?

3. How are the proportions in questions 1 and 2 different? What does this show about the situations?
Frequently Asked Questions

Q: What is the difference between a ratio and a rate?

A: A ratio is a comparison of two quantities that are measured in the same units. A rate is a comparison of two quantities that are measured in different units.

An example of a ratio is a comparison of the heights of two students, measured in centimetres. A height comparison is 140 cm to 110 cm. Since both measurements are in centimetres, the ratio can be written as $\frac{140}{110}$ or $\frac{140}{110}$.

An example of a rate is the number of words a student can type in a certain number of minutes. A typing rate is $\frac{160 \text{ words}}{2 \text{ minutes}}$, or 80 words per minute.

Q: What is a proportion?

A: A proportion is an equation that shows two equivalent ratios or rates. Multiplying or dividing both terms in one ratio or rate by the same number produces an equivalent ratio or rate.

For example, $\frac{3}{4} = \frac{21}{28}$ is a proportion. Multiplying both terms in the first ratio by 7 results in the second ratio.

Q: How do you solve a proportion with a missing term?

A: You determine the missing term using what you know about equivalent ratios. For example, how do you solve the proportion $\frac{24}{48} = \frac{6}{?}$

Method 1
Since 48 divided by 8 is 6, the number you are looking for (the missing term) must be 24 divided by 8.

$\frac{24 \div 8}{48 \div 8} = \frac{6}{6}$

The equivalent ratio is $\frac{3}{6}$. The missing term is 3.

Method 2
Since 24 is half of 48, the missing term must be half of 6, and $6 \div 2 = 3$. The missing term is 3.
Practice Questions

(2.2) 1. Write three equivalent ratios for each ratio.
   a) 5 to 6  
   b) 28:42  
   c) \( \frac{36}{45} \)

(2.2) 2. Write each comparison as a ratio. Remember that the units must be the same.
   a) 40 cm to 3 m  
   b) 100 min to 2 h  
   c) 50 g to 1 kg

(2.2) 3. Calculate the missing term in each proportion.
   a) \( \frac{3}{4} = \frac{\square}{16} \)
   b) \( 12: \square = 60:20 \)
   c) \( \frac{30}{48} = \frac{\square}{20} \)
   d) \( \frac{4}{12} = \frac{\square}{90} \)
   e) \( 2:20 = \square:50 \)
   f) \( \frac{8}{24} = \frac{26}{\square} \)

(2.2) 4. Calculate the actual length of the boat using this scale drawing.
   Scale 1:1000

(2.2) 5. On a sunny day, a streetlight that is 9 m high casts a shadow that is 6 m long. At the same time, a fence post casts a shadow that is 2 m long. How tall is the fence post?

(2.2) 6. A quarterback on a professional football team completes, on average, 10 passes for every 15 passes he attempts. How many passes can he expect to complete if he attempts 525 passes in a season? (2.2)

7. Determine whether each comparison is a ratio or a rate. Explain your thinking.
   a) Peter is 145 cm tall and Mei is 125 cm tall.
   b) 25 pens were on sale for $5.00.
   c) Kaj walked 6 km in 2 h. (2.3)

8. Christine has a summer job cutting grass. She earns $400 every 2 weeks. How much will she earn over the summer holidays, which are 12 weeks long? (2.3)

9. Rohan cycled 30 km in 2 h. How long will he take to cycle 45 km if he continues at the same rate? (2.3)

10. 10 kg of potatoes cost $4.50. How much will 50 kg of potatoes cost at this rate? (2.3)

11. A one-week summer basketball camp promises 3 coaches for every 24 players. If 276 players register for the camp, how many coaches should the camp hire? Explain how you interpreted the calculation to decide. (2.4)
Tynessa and Chang know that the ratio of male snowboarders to all snowboarders in Canada is 3:4. They want to express this relationship using a percent.

How do you rename a ratio as a percent?

A. Does the snowboarding ratio, 3:4, compare a part to a whole or a part to another part? Explain.

B. Write a proportion that relates the ratio 3:4 to an equivalent ratio out of 100.

C. Solve the proportion to find the percent that is equivalent to 3:4.

D. What fraction represents the part of all snowboarders that is male?

E. What decimal represents the part of all snowboarders that is male?

Reflecting

1. How is relating a ratio to a percent the same as solving a proportion?

2. Why can you write the ratio of male snowboarders to all snowboarders as the fraction \( \frac{3}{4} \)?

3. How do you calculate the percent that corresponds to a given fraction?

4. How do you calculate the percent that corresponds to a given decimal for tenths or hundredths?
Work with the Math

Example 1: Calculating a percent

A group of 20 Grade 7 students were surveyed about their favourite types of music. This table shows the results of the survey. What percent of students do not prefer rap?

<table>
<thead>
<tr>
<th>Type of music</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>country</td>
<td>1</td>
</tr>
<tr>
<td>rock</td>
<td>4</td>
</tr>
<tr>
<td>hip-hop</td>
<td>6</td>
</tr>
<tr>
<td>rap</td>
<td>9</td>
</tr>
</tbody>
</table>

Bonnie’s Solution: Using a grid

First I drew a grid of 20 squares, since 20 students were surveyed. I shaded 11 squares for the people who prefer country, rock, and hip-hop music.

Then I drew a grid of 100 squares. I figured out that 5 groups of 20 fits in my second grid. Each group of 20 has 11 squares shaded. I counted all the shaded squares and got 55.

\[
\frac{55}{100} = 55\%, \text{ so this is the percent of students who do not prefer rap music.}
\]

Miguel’s Solution: Using a proportion

\[
\frac{11}{20} = \frac{x}{100}
\]

I added up the number of students who do not prefer rap and wrote the ratio of this number to the total number surveyed.

I know that a percent is a ratio out of 100, so I wrote a proportion.

\[
\frac{11 \times 5}{20 \times 5} = \frac{x}{100}
\]

I know that 20 \(\times\) 5 = 100, so 11 \(\times\) 5 = \(\bigcirc\) is the number I’m looking for. 11 \(\times\) 5 = 55

So, 55\% of the students surveyed do not prefer rap music.

Example 2: Renaming a ratio

Express the red part as a ratio, a percent, a decimal, and a fraction.

Fawn’s Solution

I can see that 3 out of 5 squares are red.

I know that a percent is always out of 100, so I wrote a proportion using the ratio 3:5 and the ratio out of 100.

The scale factor that relates the two ratios is 20 because 5 \(\times\) 20 = 100.

The red squares represent 60\% of the whole shape.

Also, 60\% = \(\frac{60}{100}\), which is 0.60 or 0.6, and \(\frac{6}{10} = \frac{3}{5}\).
**A Checking**

5. What percent of each figure is shaded?
   a)  
   ![Shaded Figure]
   b)  
   ![Shaded Figure]

6. Suki is a goalie for her hockey team. During the last game, she made 36 saves out of 40 shots. Write this ratio as a percent.

7. Copy the table. Determine the missing equivalent fraction, ratio, decimal, or percent in each row.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Ratio</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (\frac{1}{2})</td>
<td>5:10</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>2:5</td>
<td></td>
<td>40%</td>
</tr>
<tr>
<td>c) (\frac{21}{100})</td>
<td></td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>d)</td>
<td></td>
<td></td>
<td>25%</td>
</tr>
</tbody>
</table>

**B Practising**

8. Look at the figures in question 5.
   a) What percent of each figure is not shaded?
   b) Write each percent in part (a) as a decimal.

9. Complete each calculation.
   a) \(\frac{7}{20} = \frac{\phantom{0}}{100}\) or \(\phantom{0}\)%
   b) \(\frac{23}{50} = \frac{\phantom{0}}{100}\) or \(\phantom{0}\)%
   c) \(\frac{19}{25} = \frac{\phantom{0}}{100}\) or \(\phantom{0}\)%
   d) \(\frac{4}{5} = \frac{\phantom{0}}{100}\) or \(\phantom{0}\)%
   e) \(\frac{3}{4} = \frac{\phantom{0}}{100}\) or \(\phantom{0}\)%

10. Write each fraction as a percent.
    a) \(\frac{32}{100}\)  
    b) \(\frac{48}{50}\)  
    c) \(\frac{16}{25}\)
    d) \(\frac{6}{10}\)
    e) \(\frac{2}{5}\)
    f) \(\frac{1}{4}\)

11. The average rainfall for a region is 25 cm. If 15 cm of rain has fallen, what percent of the average rainfall has fallen?

12. a) Write each percent as a decimal.
    b) Write each percent as a ratio.
    i) 15%  iii) 76%  v) 98%
    ii) 32%  iv) 7%  vi) 80%

13. Arrange in order from greatest to least.
    a) 74%, 32%, 45%, 66%
    b) 45%, 0.13, 0.68, 36%
    c) \(\frac{2}{8}\), 22%, \(\frac{3}{4}\), 0.58, 79%, 0.06
    d) \(\frac{12}{48}\), 62%, \(\frac{36}{60}\), 0.28, 97%, 0.46

14. Sales tax is an extra amount charged on an item, and it is expressed as a percent. Sales tax differs in the provinces. Suppose that you are buying a $1 item. How much sales tax would you pay in each province?
    a) Ontario, with a sales tax of 8%
    b) Manitoba, with a sales tax of 7%
    c) Saskatchewan, with a sales tax of 6%

15. Andrew, Mohammed, Ian, and Tyrone receive marks of \(\frac{23}{25}\), \(\frac{38}{40}\), \(\frac{27}{30}\), and \(\frac{34}{50}\), respectively. Express their marks as percents. Then arrange their marks in order from greatest to least.
16. Milk is classified according to its fat content. One type of milk has a fat content ratio of 1 part to 50 parts. Express this ratio as a percent.

17. Lindsay won 85% of her tennis matches. What is her ratio of losses to wins?

18. A basketball team won 48 out of the 80 games they played over the season.
   a) What percent of the games did they win?
   b) If they lost 30% of their games, what percent of the games did they tie?

C Extending

19. A movie poster was selling for $5.00. Its price was increased to $7.00. By what percent was its price increased?

20. Canada’s highest mountain is Mt. Logan in the Yukon Territory. The world’s highest mountain is Mt. Everest in Nepal. Mt. Logan is 5951 m high and Mt. Everest is 8848 m high.
   a) What is the ratio of the height of Mt. Logan to the height of Mt. Everest?
   b) Determine the decimal equivalent of this ratio. Round your decimal to the nearest hundredth.
   c) What percent of the height of Mt. Everest is the height of Mt. Logan?

21. a) When \( \frac{1}{3} \) is written as a percent such as \( \% \), why is \( \% \) not a whole number?
   b) Name a different ratio for a part to a whole that is not equivalent to a whole number percent. Explain how you know.

Mental Math

MULTIPLYING BY TENTHS AND HUNDREDTHS

Look at the number patterns.

\[
\begin{align*}
34 \times 100 &= 3400 \\
34 \times 10 &= 340 \\
34 \times 1 &= 34 \\
34 \times 0.1 &= 3.4 \\
34 \times 0.01 &= 0.34 \\
34 \times 200 &= 6800 \\
34 \times 20 &= 680 \\
34 \times 2 &= 68 \\
34 \times 0.2 &= 6.8 \\
34 \times 0.02 &= 0.68 \\
\end{align*}
\]

1. How can you use mental math to multiply a whole number by 0.1? by 0.2?

2. How can you use mental math to multiply a whole number by 0.01? by 0.02?

3. To calculate each product using mental math, multiply the whole number by 0.1 or 0.01 first.
   a) \( 0.2 \times 16 \)  d) \( 0.3 \times 15 \)  g) \( 0.03 \times 120 \)
   b) \( 0.7 \times 300 \)  e) \( 0.2 \times 7500 \)  h) \( 0.04 \times 240 \)
   c) \( 0.4 \times 25 \)  f) \( 0.04 \times 21 \)  i) \( 0.02 \times 408 \)
Brian and his family went out to dinner to celebrate his sister’s birthday. Their bill came to $80.00. It is customary to leave a 15% tip for the server.

What amount of tip should Brian’s family leave for the server?

A. What is 10% of $80.00?
B. What fraction of 10% is 5%?
C. What is 5% of $80.00? Explain how you found this.
D. Determine the amount of the tip.
E. You could have solved \( \frac{15}{100} = \frac{3}{80} \) to calculate the tip. Explain why. Calculate the value of the missing term.

Reflecting

1. Suppose that the bill at the restaurant had been $76.35.
   a) Why would some people round the amount to $80 before calculating the tip?
   b) Would you use this method if you had a calculator? Explain your answer.
2. What scale factor did you use to solve the proportion in step E?
3. In step E, you used a proportion to calculate the tip. In steps A to D, you did not use a proportion. Compare the strategies.
4. Explain why a 15% tip is easier to calculate than a 13% tip.
Work with the Math

Example 1: Calculating with percent

The largest oil spill in North America occurred in Prince William Sound, Alaska. Close to 42 million litres of oil were spilled. Clean-up crews were able to recover 14% of the spill. About how many litres of oil were recovered?

Romona’s Solution: Solving a proportion

\[
\frac{\text{amount recovered (millions of litres)}}{\text{amount spilled (millions of litres)}} = \frac{14}{100} = \frac{\text{(unknown) million}}{42}
\]

I wrote a proportion that relates the amount of oil spilled to the percent of oil recovered.

I found the scale factor that relates the two ratios.\[42 \div 100 = 0.42 \text{ so } 100 \times 0.42 = 42\]

I used the scale factor and a calculator to calculate the missing term in the proportion.

About 5.88 million litres of oil were recovered.

Paul’s Solution: Multiplying by an equivalent decimal

\[	ext{amount spilled} = 14\% \text{ of } 42 \text{ million}
\]

\[14\% = \frac{14}{100} \text{ or } 0.14\]

\[	ext{amount spilled} = 0.14 \times 42 = 5.88
\]

The decimal form of 14% is 0.14. So, I can calculate 14% of 42 by multiplying 0.14 \times 42 on my calculator.

So, 5.88 million litres of oil were recovered.

Example 2: Calculating a percent

In the student council election, Julie received 168 votes out of 240 votes. Determine the percent of the votes she received.

Bonnie’s Solution

\[
\frac{\text{votes for Julie}}{\text{total votes}} = \frac{168}{240}
\]

I wrote the first ratio, which is the number of votes for Julie to the total number of votes for all the candidates.

Since percent means out of 100, I used \[\square\] to represent the number of votes that Julie got out of 100.

I divided 240 by 100 to determine the scale factor. The scale factor is 2.4 because \[240 \div 2.4 = 100\].

I divided 168 by the scale factor using my calculator.

Julie got 70% of the votes.
Example 3: Calculating a number from a percent

There are 10 boys in Erin’s music class. If 40% of the students in the music class are boys, how many students are in the music class?

**Miguel’s Solution: Using logical reasoning**

Percent of class that are girls = 100% – percent that are boys
= 100% – 40%
= 60%

Number of girls = 60% of class
= 40% of class + 20% of class
= 10 + 5
= 15

Total number of students = 10 + 15
= 25

**Fawn’s Solution: Solving a proportion**

\[
\frac{10}{\text{total}} = \frac{40}{100}
\]

I used “total” to represent the total number of students in the class. Since percent means out of 100, I know that 40% means 40 boys out of 100. I wrote a proportion.

\[
\frac{10}{\text{total}} = \frac{40 \div 4}{100 \div 4}
\]

I figured out that 40 ÷ 4 equals 10, so the total must be 100 ÷ 4.

\[
100 \div 4 = 25
\]

I divided and determined that the total is 25. There are 25 students in the music class.

**Checking**

5. Calculate 15% of 60 using each method.
   a) Use a mental calculation. Give only the answer.
   b) Use a proportion.
   c) Multiply by the equivalent decimal.

6. Determine each number.
   a) 25% of 60 = 15
   b) 10% of 40 = 4
   c) 20% of 45 = 9
   d) 12% of 54 = 6

**Practising**

7. Calculate.
   a) 50% of 20
   b) 75% of 24
   c) 20% of 45
   d) 12% of 50
   e) 15% of 200
   f) 44% of 250

8. Calculate.
   a) 50% of \( \square = 15 \)
   b) 25% of \( \square = 22 \)
   c) 10% of \( \square = 7 \)
   d) 75% of \( \square = 12 \)
   e) 15% of \( \square = 24 \)
   f) 44% of \( \square = 110 \)
9. Out of a batch of 600 computers, 30 failed to pass inspection due to faulty wiring. What percent failed to pass inspection?

10. There are 12 girls with blond hair in Katya’s gymnastics class. This is 25% of the entire class.
   a) Use a mental calculation to determine the total number of students in the gymnastics class. Record only the answer.
   b) Use a proportion to determine the total number of students in the class.

11. A dealer paid $6000 for a used car. The dealer wants to make a profit that is 25% of the price he paid for the car.
   a) What profit does the dealer want to make?
   b) How much should the dealer sell the car for?

12. If Sarah’s mother really likes the service in a restaurant, she leaves a 20% tip. Their last bill was $110.00, and Sarah’s mother left $22.00 for the tip. Explain how you know that this is 20%.

13. To sell a new chocolate bar, the manufacturer advertises “20% MORE, FOR FREE!” If a standard chocolate bar is 50 g, what size is the new bar?

14. A roast-beef sandwich contains 18 g of fat, and a slice of pizza contains 25 g of fat. If you choose to eat pizza at lunch instead of a roast-beef sandwich, what percent greater is your fat intake? Round to the nearest whole number percent.

15. Suki buys a dress on sale for 20% off the original price. She saves $40.
   a) What was the original price of the dress?
   b) How much did Suki pay for the dress?

16. A pair of jeans usually costs $80.00. The jeans are on sale at Jane’s Jean Shop for 50% off. Denim Discounters offers the same jeans at 30% off, as well as a further 20% off the already discounted price. Are the jeans the same price at both stores? Justify your answer.

17. Use the circle graph to estimate the number of hours that Matthew spends on each activity in a 24 h day.

18. Last year, Jasleen was 150 cm tall. This year, her doctor measures her height and tells her that she has grown 20%. How many centimetres has she grown?

Extending

19. A new process in a factory has increased production by 12%. If workers are now producing 30 more units per day, how many units did they produce per day before the new process was introduced?

20. The tax rate for businesses was increased by half of 1%. If a business was paying $20 000 in taxes before the increase, how much would the business pay after the increase?

21. Anthony had a 12 cm by 15 cm photo enlarged. If the dimensions of the enlargement are 24 cm by 30 cm, by what percent was the area of the photo enlarged?
Romona says, “I have a poster that is 0.5 m by 1.1 m. I want a copy to put above my desk. I’m going to change each dimension to 60% of its original size.”

Bonnie says, “60% is \( \frac{60}{100} \). That’s the same as \( \frac{6}{10} \) or 0.6. You can multiply \( 0.6 \times 0.5 \) to calculate 60% of the width.”

**What is the width of the reduced copy?**

**Example 1: Using a grid to model multiplication with decimals**

Use a 10-by-10 grid to determine the product of \( 0.6 \times 0.5 \).

**Romona’s Solution**

I coloured 5 columns of squares red on a 10-by-10 grid to represent 0.5.

I coloured 6 rows of squares blue to represent 60% of the grid.

There are 30 purple squares in the overlap. The purple squares represent \( \frac{6}{10} \) of \( \frac{5}{10} \), which is \( \frac{30}{100} \) of the 10-by-10 grid.

So, \( 0.6 \times 0.5 = 0.30 \).
Reflecting

1. Why will Romona’s strategy of using a 10-by-10 grid to model multiplying tenths by tenths always have an answer in hundredths?

2. a) How did Bonnie’s method allow her to use what she already knew about whole number multiplication?
   b) Why did Bonnie need to divide by 10 after she multiplied by 10?

3. a) How did estimating help Paul check his answer?
   b) Why did Paul need to divide by 100 after he multiplied by 100?

4. The length of the poster is 1.1 m. Explain how to calculate 60% of the length.
Example 4: Estimating and calculating distance travelled

Stephen can ride his bike at an average speed of 15.5 km/h. At this speed, how far will he travel in each amount of time?

a) 30 min  

b) 0.75 h

Solution

a) Estimate.

30 min is 0.5 h or half an hour. In one hour, Stephen travels 15.5 km, so he’d travel about 8 km in half an hour.

b) Estimate.

0.75 h is a bit less than 1 h. So Stephen should travel a bit less than 15.5 km—maybe 13 km.

Calculate.

Distance = speed × time, so the calculation is 15.5 × 0.5.
Multiply by 10. \(15.5 \times (0.5 \times 10) = 15.5 \times 5\)
\[= 77.5\]
Divide by 10. \(77.5 \div 10 = 7.75\) or 7.8, rounded to one decimal place
In 0.5 h, Stephen can ride 7.8 km at 15.5 km/h.

Calculate.

Distance = speed × time, so the calculation is 15.5 × 0.75.
Multiply by 100. \(15.5 \times (0.75 \times 100) = 15.5 \times 75\)
\[= 1162.5\]
Divide by 100. \(1162.5 \div 100 = 11.625\) or 11.6, rounded to one decimal place
In 0.75 h, Stephen can ride 11.6 km at 15.5 km/h.

A Checking

5. Use a 10-by-10 grid to model, and then calculate.
   a) \(0.3 \times 0.8\)  
   b) \(0.2 \times 0.7\)

6. Estimate each product.
   a) \(9.7 \times 0.63\)  
   b) \(3.75 \times 5.86\)

7. Choose a strategy and multiply.
   a) \(0.2 \times 3.4\)  
   b) \(0.8 \times 7.59\)

B Practising

8. Use a 10-by-10 grid to model, and then calculate.
   a) \(0.2 \times 0.6\)  
   b) \(0.8 \times 0.7\)

9. Estimate, and then calculate. Show the strategy you used.
   a) \(0.8 \times 1.3\)  
   d) \(2.6 \times 1.01\)
   b) \(1.1 \times 2.3\)  
   e) \(0.02 \times 1.5\)
   c) \(2.2 \times 0.03\)  
   f) \(0.35 \times 10.1\)
10. Explain why you can multiply $0.5 \times 0.64$ mentally more easily than $0.7 \times 0.64$.

11. How would you place the digits 6, 7, and 8 so that the product is as close to 5 as possible? 

12. Calculate the distance a car will travel in 3.5 h if its speed averages 92.5 km/h.

13. A store advertises hamburger meat for sale at $2.25 /kg. How much will 3.4 kg cost?

14. A package of 12 guitar picks costs $5.04. At that price, how much should 18 picks cost?

15. A Japanese bullet train recently maintained an average speed of 317.5 km/h for a trip that lasted 3 h 15 min. Calculate the distance covered by the train on that trip.

16. Gasoline recently cost 82.5¢/L. If Tonya’s car holds 58.5 L, how much should it cost to fill the tank?

17. What is the area of a rectangle that is 3.2 m wide and 5.1 m long?

18. Suki wants her bedroom ceiling painted. The room is a rectangle with dimensions 4.2 m by 3.9 m. The label on the paint can says one can has enough paint to cover 12 m². Explain whether one can is enough for Suki to put two coats of paint on the ceiling.

19. Richard works at a clothing store. He is paid $150 a week plus 9% of the value of his sales for the week. Last week his sales totalled $457.85. Calculate Richard’s earnings for the week.

20. The adult height of a male is about 1.19 times his height at age 12. The adult height of a female is about 1.07 times her height at age 12. Predict how tall Miguel and Romona will be as adults if Miguel is 1.5 m and Romona is 1.6 m.

Extending

21. According to Statistics Canada, the 2002 birth rate for the country was 10.5 births for every 1000 people. In July 2002, the Canadian population was 31 361 611. Calculate the approximate number of births that took place in Canada in 2002.

22. The Information About Canada Web site indicates that about 79% of the population lives in cities. Use the July 2002 population data in question 21 to calculate the approximate number of city dwellers in Canada.

23. A used car is listed for sale at $9000. The price of the car is reduced by 20% for quick sale. Then it is reduced by 20% of the sale price. What is the final sale price of the car?

24. Meagan said that to multiply $1.3 \times 1.3$, you can multiply $1 \times 1$ and $0.3 \times 0.3$ and the answer is 1.09. Do you agree? Explain.
Chapter 2

2.8 Decimal Division

GOAL
Use decimal division to solve ratio and rate problems.

Learn about the Math

Paul is selling used comic books. Miguel has $2.50 to buy some.

How many comic books can Miguel buy with $2.50?

Example 1: Using grids to model division with decimals

Comic books cost $0.45 each. Use 10-by-10 grids to determine how many comic books you can buy with $2.50. How much money will be left over?

Miguel’s Solution

I used three 10-by-10 grids to represent the $2.50.

Each group of 45 squares represents the price of one comic book.

There are 5 groups of 45 squares and 25 squares left over.

I can buy 5 comic books and I’ll have 25¢ left over.
Example 2: Dividing hundredths using equivalent fractions

Calculate $2.50 \div 0.45$ to figure out how many comic books Miguel can buy.

**Paul's Solution**

\[
\begin{align*}
&2.50 \div 0.45 \\
= &\quad \quad 2.50 \\
0.45 &\quad \quad \text{I know that } 1 \div 4 \text{ means } \frac{1}{4}, \text{ so I can write } 2.50 \div 0.45 \text{ as } \frac{2.50}{0.45}. \\
= &\quad \quad 2.50 \times 100 \\
0.45 &\quad \quad \text{If I multiply } 0.45 \text{ by } 100 \text{ to get a whole number, I need to multiply } 2.50 \text{ by } 100. \\
= &\quad \quad \frac{250}{45} \\
= &\quad \quad 5 \\
&\quad \quad \frac{250}{45} \div \frac{225}{25} \\
\text{The quotient is } &5 \text{ and the remainder is } 25. \\
&\quad \quad \text{Miguel can buy } 5 \text{ comic books and have } 25\text{¢ left over.}
\end{align*}
\]

Example 3: Dividing tenths using equivalent fractions

Estimate $91.8 \div 3.4$. Then divide.

**Fawn's Solution**

**Estimate.** $91.8 \div 3.4$ is about $90 \div 3 = 30$.

**Calculate.**

\[
\begin{align*}
&91.8 \div 3.4 \\
= &\quad \quad 91.8 \\
3.4 &\quad \quad \text{I rounded } 3.4 \text{ to } 3. \text{ I rounded } 91.8 \text{ to } 90 \text{ because } 90 \text{ is a multiple of } 3. \\
= &\quad \quad 91.8 \times 10 \\
3.4 \times 10 &\quad \quad \text{I can write } 91.8 \div 3.4 \text{ as } \frac{91.8}{3.4}. \\
= &\quad \quad \frac{918}{34} \\
= &\quad \quad \frac{918}{34} \div \frac{238}{0} \\
\text{The quotient is } &27. \\
&\quad \quad \text{I know that my answer is reasonable because } 27 \text{ is almost } 30.
\end{align*}
\]

Reflecting

1. When would Miguel’s strategy of using 10-by-10 grids result in a whole number quotient without a remainder?

2. Why can Paul write $2.50 \div 0.45$ as $\frac{2.50}{0.45}$?

3. a) Why did Fawn choose 10 as the multiplier?  
   b) Why is it necessary to multiply both the divisor and the dividend by the same number?

4. How did estimating help Fawn check her answer?
Example 4: Dividing using measurements

Mei has 5 m of ribbon. She wants to divide it into equal pieces.

a) How many pieces will there be if each piece is 25 cm long?

b) What if each piece is 0.8 m long?

Solution

\[
\begin{align*}
a) \quad 5 \div 0.25 &= \frac{5}{0.25} \\
&= \frac{5 \times 100}{0.25 \times 100} \\
&= \frac{500}{25} \\
&= 20 \\
&= 25 \div 500 \\
&= 0
\end{align*}
\]

Change the measurements so that the units are the same.
25 cm = 0.25 m
Multiply the divisor 0.25 by 100 to get a whole number.
Then multiply the dividend 5 by 100.
There will be 20 pieces, each 25 cm long, with no ribbon left over.

\[
\begin{align*}
b) \quad 5 \div 0.8 &= \frac{5}{0.8} \\
&= \frac{5 \times 10}{0.8 \times 10} \\
&= \frac{50}{8} \\
&= 6 \text{ R } 2
\end{align*}
\]

Multiply the divisor 0.8 by 10 to get a whole number. Then multiply the dividend 5 by 10.
There will be 6 pieces, each 0.8 m long, with some ribbon left over.

A Checking

5. Use 10-by-10 grids to model, and then calculate.
   a) \(2.7 \div 0.9\)
   b) \(3.6 \div 0.18\)
   c) \(12.4 \div 0.4\)

6. Estimate each quotient.
   a) \(3.6 \div 0.9\)
   b) \(7.8 \div 1.3\)

7. Choose a strategy, then divide.
   a) \(2.7 \div 0.03\)
   b) \(4.59 \div 0.9\)
   c) \(0.25 \div 0.04\)

B Practising

8. Use 10-by-10 grids to model, and then calculate.
   a) \(3.6 \div 0.4\)
   b) \(2.8 \div 0.14\)
   c) \(2.25 \div 0.15\)

   a) \(2.7 \div 0.4\)
   b) \(3.13 \div 0.02\)
   c) \(10.2 \div 1.5\)
   d) \(0.27 \div 0.04\)
   e) \(14.4 \div 0.12\)
   f) \(0.04 \div 0.02\)
10. Use division to show the number of coins you would have if you had $11.50 in each type of coin.
   a) dimes       c) quarters
   b) nickels     d) pennies

11. Why is the result of \( 1.25 \div 0.01 \) the same as \( 1.25 \times 100 \)?

12. Explain how you can tell without actually calculating that the remainder for \( 2.95 \div 0.05 \) is 0, but the remainder for \( 2.95 \div 0.02 \) is not 0.

13. Nathan has 11.4 m of rope. He wants to divide it into equal pieces. How many pieces will there be if the pieces are these lengths?
   a) 80 cm long       c) 0.7 m long
   b) 1.4 m long       d) half a metre

14. To the nearest hour, how long will it take to walk 10 km at each speed?
   a) 4.5 km/h       b) 3.2 km/h

15. How many 0.35 L glasses can be filled from a 1.5 L bottle of water?

16. To the nearest litre, how much gasoline can you buy with $20.00 if the price for gas is 87.5¢/L?

17. Susan earned $191.25 last week. She is paid $8.50/h. How many hours did she work?

18. Calculate the average speed of a train that completed a 525 km trip in 4.7 h. Round your answer to the nearest whole number.

19. The adult height of a male is about 1.19 times his height at age 12. The adult height of a female is about 1.07 times her height at age 12. Predict how tall each of these people were when they were 12.
   a) an adult male 1.8 m tall
   b) an adult female 1.8 m tall

20. Kyle is filling his brother’s wading pool. The pool holds 180 L of water and the hose supplies water at 22.5 L/min. To the nearest minute, how long will it take to fill the pool?

21. When water freezes, its volume increases. For example, when 100 cm\(^3\) of water is frozen, about 109 cm\(^3\) of ice results.
   a) What volume of ice will result from freezing 12 cm\(^3\) of water?
   b) What volume of water must be frozen to produce 100 cm\(^3\) of ice?

22. Snails move at approximately 0.013 m/s. How long would it take a snail moving at this speed to travel the 450 km distance from Toronto to Ottawa?

23. Order the times required for each trip from least to greatest.

<table>
<thead>
<tr>
<th></th>
<th>Distance</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bullet train</td>
<td>571.5 km</td>
<td>317.5 km/h</td>
</tr>
<tr>
<td>Airplane</td>
<td>1191 km</td>
<td>680.5 km/h</td>
</tr>
<tr>
<td>Car</td>
<td>204 m</td>
<td>90 km/h</td>
</tr>
</tbody>
</table>
Tape a line on the floor, 3 m from your basket. Create a table like the one below to record your team’s results.

Number of players: 4 per team

Rules
1. Each player on your team takes 10 shots. Record the “Shots in basket” in your table.
2. Calculate the “Percent in basket” for each player on your team. Record this in your table.
3. Calculate your team’s “Percent in basket” for the 40 shots taken.
4. Compare your team’s results with the results for the other teams in your class. The team with the highest “Percent in basket” is the champion.
5. What is the “Percent in basket” for your class?

Optional: Challenge other classes in your school to see which class is the “Wastepaper Basketball” champion.

You will need
• a spongy ball or a crumpled piece of paper
• a measuring tape or a metre stick
• a wastepaper basket or another suitable “basket” (such as a box or a bucket)
• tape
• a calculator

<table>
<thead>
<tr>
<th>Player</th>
<th>Shots in basket</th>
<th>Shots taken</th>
<th>Ratio of shots in basket to shots taken</th>
<th>Percent in basket</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. a) Write the ratio of white squares to green squares.
    b) What fraction of the squares are white?
    c) What percent of the squares are green?

2. Determine three ratios that are equivalent to 5:9.

3. Calculate the missing term in each proportion.
   a) \( \frac{2}{11} = \frac{?}{55} \)
   c) \( \frac{9}{?} = \frac{21}{28} \)
   b) \( \frac{36}{42} = \frac{12}{?} \)
   d) \( \frac{6}{9} = \frac{?}{15} \)

4. The height of a building in a scale drawing is 9 cm. The scale is 1:600. Explain how you would use the scale to find the actual height of the building.

5. Two large cups of coffee cost $4.00. How much money will Marianne need if she wants to buy nine large cups of coffee?

6. Nicole earned $72 in 9 h. At this rate, how much would she earn in 12 h?

7. The air you breathe is \( \frac{1}{5} \) oxygen. What percent of the air is made up of other gases?

8. Copy and complete the table to express the equivalent forms in each row.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Ratio</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td></td>
<td>21:60</td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td></td>
<td></td>
<td>0.18</td>
</tr>
<tr>
<td>c)</td>
<td></td>
<td>0.18</td>
<td>82%</td>
</tr>
</tbody>
</table>

9. Calculate.
   a) 40% of 35 is \(?\).
   b) 21 out of 30 is \(?\)%.
   c) 38% of \(?\) is 95.

10. At a baseball game, the stadium is 65% full. If the stadium’s capacity is 1800, how many people are at the game?

11. Braydon buys a new CD and pays 8% sales tax. If the sales tax is $1.20, calculate the price of the CD.

12. Akeem bought a used video game for $38. Later, he sold it for 40% less than he paid. For how much did Akeem sell the video game?

13. The ratio of children to adults at an art show is 22:18.
   a) What percent of the crowd are children?
   b) What percent of the crowd are adults?

14. Why is calculating 10% of an amount easier than calculating 17% of the amount?

15. How is calculating 1% of an amount similar to calculating 10% of the amount?

16. Why might you calculate 50% of 212 in a different way than you would calculate 42% of 212?

17. How do you know that each answer is about 5?
   a) \( 1.4 \times 3.5 \)
   b) \( 0.7 \times 7.34 \)
   c) \( 65.2 \div 12.9 \)
   d) \( 2.46 \div 0.53 \)

18. How much greater is \( 27.9 \times 4.5 \) than \( 27.9 \div 4.5 \)?
Frequently Asked Questions

Q: What is a percent?
A: A percent is a ratio out of 100. A percent is used to compare a part to a whole, so the second term refers to a whole or a total. To determine a percent from a ratio, calculate the equivalent ratio out of 100.

For example,
\[
\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = \frac{2}{5} = 40\%.
\]

Q: How are ratios, fractions, decimals, and percents related?
A: If you are given one form (ratio, fraction, decimal, or percent), you can determine the other three equivalent forms. For example, 4:5 is \(\frac{4}{5}\) which means 80 out of 100 or 80\%, and 80 hundredths = 0.80 or 0.8, which is \(\frac{4}{5}\).

Q: How do you solve a problem that involves percent?
A: Depending on the problem and numbers involved, you could draw a diagram, use the relationship between the numbers, or solve a proportion by relating to a ratio out of 100.

Q: How can you multiply or divide with two decimals?
A: You can use a model or you can use powers of 10.

For multiplication, multiply either factor by a power of 10 to get a whole number, multiply this whole number by the other factor, then divide the result by the same power of 10. For 0.3 \(\times\) 4.67,

\[
0.3 \times 10 = 3, \text{ then } 3 \times 4.67 = 14.01,
\]
and 14.01 \(\div\) 10 = 1.401.

For division, multiply the divisor by a power of 10 to get a whole number, multiply the dividend by the same power of 10, then divide. For 227.8 \(\div\) 3.4,

\[
\text{write } \frac{227.8}{3.4}, \text{ then } \frac{227.8 \times 10}{3.4 \times 10} = \frac{2278}{34},
\]
and 2278 \(\div\) 34 = 67.
1. Write two equivalent ratios for each ratio.
   a) 9 : 20
   b) \( \frac{4}{5} \)
   c) 21 to 3

2. Express both quantities in each comparison using the same units. Write the comparison as a ratio in fraction form. Then write an equivalent ratio using whole numbers.
   a) 85¢ to $1.20
   b) 24 kg to 80 g

3. Determine each missing term.
   a) \( \frac{2}{7} = \frac{\square}{21} \)
   b) \( \frac{36}{9} = \frac{72}{\square} \)
   c) \( 4 : \square = 8 : 14 \)
   d) \( \frac{2}{12} = \frac{\square}{48} \)

4. The average height of an ostrich is 255 cm. Suppose that you want to make a scale drawing of an ostrich. The scale is 30 : 1. What height will you make the ostrich in your drawing? Explain your thinking.

5. A car travels 180 km in 3 h. At this rate, how far will the car travel in 5 h?

6. Copy and complete the table to express the equivalent forms in each row.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Ratio</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 24:30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 11/3</td>
<td>0.36</td>
<td>86%</td>
<td></td>
</tr>
<tr>
<td>d) ( \frac{3}{3} )</td>
<td></td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>e)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. The ratio of red cars to black cars in the school parking lot is 6:8. Vanessa determines that \( \frac{6}{8} = 75\% \). She concludes that 75% of the cars in the parking lot are red. Is she correct? Explain.

8. The following circle graph shows the ingredients that are used to make a sausage and mushroom pizza. The percent of each ingredient, by weight, is given.
   a) Determine the fraction, by weight, of each ingredient.
   b) Determine the decimal, by weight, of each ingredient.
   c) If you used fractions or decimals to represent the data, would the shape of the circle graph change? Explain. (2.5)

9. Determine each missing number. (2.6)
   a) 25% of 84 = \( \square \)
   b) 24% of 200 = \( \square \)
   c) 10% of \( \square \) = 5

10. A movie theatre has sold 75% of its seats for the 7:00 p.m. show. If the theatre has 440 seats, how many tickets are sold? (2.6)

11. Last week, Raj earned $58. He spent $42 and saved the rest. Michael earned $83 and saved $20. Who saved the greater percent of his earnings? (2.6)

12. Draw a model to calculate. (2.7)
   a) 0.2 \times 0.9
   b) 3.2 \div 0.4

13. Calculate. (2.8)
   a) 1.4 \times 5.3
   b) 0.9 \times 3.28
   c) 6.3 \div 2.1
   d) 6.93 \div 0.33
**Ball Bounce-ability**

Have you ever dropped a ball to see how high it would bounce? Do different types of balls bounce better than others?

**How can you determine which type of ball bounces the best?**

A. With a partner, choose three different types of balls. Each ball should be a different size and material; for example, a basketball, golf ball, tennis ball, or soccer ball.

B. Create a table to record your data from steps C and D.

C. Select the first ball. One person will be the dropper, and the other person will be the measurer.
   - Find a height that is comfortable for the dropper to release the ball. Measure and record this height.
   - The dropper lets the ball fall to the floor.
   - As the ball bounces back up, the measurer notes the maximum bounce height. (This is the distance that the ball bounces up from the floor on the first bounce.)
   - The dropper then measures the distance from the floor to the maximum bounce height.

Complete 5 or 10 trials, and record the average in your data table. Use the same drop height each time.

D. Repeat step C for the other two balls.

E. Determine the ratio of bounce height : drop height for each ball.

F. For each ball, what is the ratio for comparing the bounce height with the drop height? What is the percent for this?

G. Write a short report about your findings. Discuss the results of your experiment, and rank the bounce-ability of each ball.
Rock Band Manager

Keith Porteous manages such highly successful music acts as 54/40. He lives in Vancouver, British Columbia, and has been a rock band manager for over 15 years.

Keith is paid a commission for his work. The commission is a percentage of the artists’ earnings. “The whole music business is based on percentages,” says Keith. “We call the percentages ‘points.’ If you earn 10%, you make 10 points.” How does Keith earn his points? “I talk on the phone all day,” Keith joked in a recent interview. Actually, his “talk” leads to touring and recording opportunities for the acts he represents.

Problems, Applications, and Decision Making

1. At one concert, a band earns $10,000. Keith makes 20 points. How much money does Keith earn?

2. The profit from a concert is $12,750. A manager makes 10.5 points. How much money does the manager earn?

3. Suppose that you are the manager of a rock band. You earn $3,750 for a concert. The concert takes in $25,000. How many points do you make?

4. List some advantages and disadvantages of being paid by commission.

Keith Porteous explains how the concert profit is divided: “Let’s say each concert ticket costs $15. The seating capacity of the concert hall is 1000 people. So you multiply $15 by 1000 seats. Then you multiply the product by 10%, which is the agent’s commission. What’s left over you multiply by 20%, which is the manager’s commission. The rest belongs to the artists.”

5. Write a formula to work out how much money the musicians earn.

6. Why is the order of operations important? Could the formula be written any other way? Explain.
7. How much money do the musicians earn? How much money does the manager earn? How much money does the agent earn?

8. What would happen to everyone’s earnings if the ticket price were increased to $20? Calculate the earnings.

9. Suppose that the concert is moved to a hall that holds 1200 people. Use a calculator to find the earnings of the band, the manager, and the agent if the ticket price is $15.

10. A band plays in a large stadium and earns $270 000. The manager’s share is 20% of the band’s earnings. Estimate how much money the manager earns. Check your estimate.

**Advanced Applications**

Keith explains how the money from T-shirt sales is divided: “If T-shirts are sold at a concert, the concert hall takes 15% of the total sales, before sales taxes. What’s left is called the net. The merchandisers who make and distribute our T-shirts usually have an agreement with the artists to keep 67% of the net. Out of what’s left, the manager gets 20%. The band gets the rest.”

11. Write a formula to calculate how much money the band receives from T-shirt sales.

12. If T-shirt sales are $7500, find out how much money the concert hall gets, how much money is paid in GST and PST, and how much money the merchandisers, manager, and band get. What if T-shirt sales are $12 750?

“If you think of the profit from a song as being 100%, the publishing expenses are 50% of the profit,” Keith explains. “The songwriter gets the other 50%. Of this, 50% is for composing the music and 50% is for writing the lyrics to the song. If the lyrics are co-written, the writers divide the writing portion.”

13. The publishers of a song make $100 000. How much money does the lyricist earn? How much money does the person who composed the music earn?

14. The profit from a song is $250 000. The lyrics were co-written by two people. How much money does each person earn?